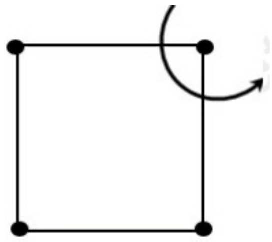
System of Particles and Rotational Motion

Question1

Four particles each of mass 1kg are placed at four corners of a square of side 2m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is____ kgm^2.



[27-Jan-2024 Shift 1]

Answer: 16

Solution:

$$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$$

$$= 4ma^2$$

$$= 4 \times 1 \times (2)^2 = 16$$

Question2

A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is 7/x where x is

[27-Jan-2024 Shift 2]

Answer: 7

Solution:

In pure rolling work done by friction is zero. Hence potential energy is converted into kinetic energy. Since initially the ring and the sphere have same potential energy, finally they will have same kinetic energy too.

∴ Ratio of kinetic energies = 1

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$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

Question3

A body of mass 5kg moving with a uniform speed $3\sqrt{2}ms^{-1}$ in $X - Y$ plane along the line $y = x + 4$. The angular momentum of the particle about the origin will be ___$kgm^2\ s^{-1}$.

[29-Jan-2024 Shift 2]

Answer: 60

Solution:

$$y - x - 4 = 0$$

d_1 is perpendicular distance of given line from origin.

$$d_1 = \left| \frac{-4}{\sqrt{1^2 + 1^2}} \right| \Rightarrow 2\sqrt{2}m$$

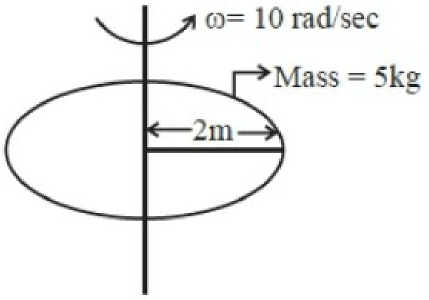
So,

$$|\vec{L}| = mvd_1 = 5 \times 3\sqrt{2} \times 2\sqrt{2}\ kg\,m^2/s$$

$$= 60\ kg\,m^2/s$$

Question4

Consider a Disc of mass 5kg, radius 2m, rotating with angular velocity of 10rad/ s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is_____ J.



[30-Jan-2024 Shift 1]

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Answer: 250

Solution:

$$\overrightarrow{L}_i = I\omega_i = \frac{MR^2}{2}\cdot\omega = 100\text{kgm}^2/s$$

$$E_i = \frac{1}{2}\cdot\frac{MR^2}{2}\cdot\omega^2 = 500\text{J}$$

$$\overrightarrow{L}_i = \overrightarrow{L}_f \Rightarrow 100 = 2I\omega_f$$

$$\omega_f = 5\text{ rad}/sec$$

$$E_f = 2\times\frac{1}{2}\cdot\frac{5(2)^2}{2}\cdot(5)^2 = 250\text{J}$$

$$\Delta E = 250\text{J}$$

Question5

Two discs of moment of inertia $I_1 = 4\text{kgm}^2$ and $I_2 = 2\text{kgm}^2$ about their central axes & normal to their planes, rotating with angular speeds 10rad/ s & 4rad/ s respectively are brought into contact face to face with their axe of rotation coincident. The loss in kinetic energy of the system in the process is ______J.

[30-Jan-2024 Shift 2]

Answer: 24

Solution:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_0 \text{ (C.O.A.M.)}$$

$$\text{gives } \omega_0 = 8\text{ rad}/s$$

$$E_1 = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 = 216\text{J}$$

$$E_2 = \frac{1}{2}(I_1 + I_2)\omega_0^2 = 192\text{J}$$

$$\therefore \Delta E = 24\text{J}$$

Question6

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Two identical spheres each of mass 2kg and radius 50cm are fixed at the ends of a light rod so that the separation between the centers is 150cm. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is x/20 kg m^2, where the value of x is___

[31-Jan-2024 Shift 2]

Answer: 53

Solution:

$$I = \left(\frac{2}{5}mR^2 + md^2 \right) \times 2$$

$$I = 2\left(\frac{2}{5} \times 2 \times \left(\frac{1}{2} \right)^2 + 2 \times \left(\frac{3}{4} \right)^2 \right) = \frac{53}{20}\,kg - m^2$$

$$X = 53$$

Question7

A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}} m/s^2$, where x =_____ (use g = 10m/ s^2).

[29-Jan-2024 Shift 1]

Solution:

For rolling motion, $a = \dfrac{g\sin\theta}{1 + \dfrac{I_{cm}}{MR^2}}$

$$a = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

$$= \frac{2 \times 10 \times \frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}}$$

Therefore $x = 10$

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Question8

A body of mass ' m ' is projected with a speed ' u ' making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of ' X ' is______

[31-Jan-2024 Shift 2]

Options:

Answer: 8

Solution:

$$L = mu\cos\theta \frac{u^2 sin^2\theta}{2g}$$

$$= mu^3 \frac{1}{4\sqrt{2}g} \Rightarrow x = 8$$

Question9

A ball of mass 0.5kg is attached to a string of length 50cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400N. The maximum possible value of angular velocity of the ball in rad/ s is,:

[1-Feb-2024 Shift 1]

Options:

A.

1600

B.

40

C.

1000

D.

20

Answer: B

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Solution:

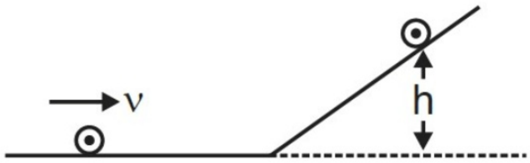
$$T = m\omega^2\ell$$

$$400 = 0.5\omega^2 \times 0.5$$

$$\omega = 40\,\text{rad}/s.$$

Question10

A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



[1-Feb-2024 Shift 2]

Options:

A.

$\frac{v^2}{g}$

B.

$\frac{3}{4}\frac{v^2}{g}$

C.

$\frac{1}{2}\frac{v^2}{g}$

D.

$\frac{2}{3}\frac{v^2}{g}$

Answer: C

Solution:

Only the translational kinetic energy of disc changes into gravitational potential energy. And rotational KE remains unchanged as there is no friction.

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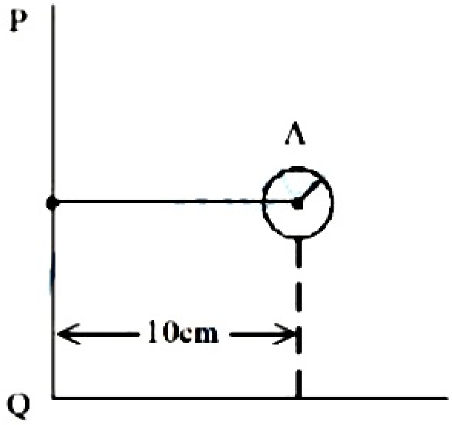
$$\frac{1}{2}mv^2 = mgh$$

$$h = \frac{v^2}{2g}$$

Question11

Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is___



[24-Jan-2023 Shift 1]

Answer: 110

Solution:

$I_{PQ} = I_{cm} + md^2$

$I_{PQ} = \frac{2}{5}mR^2 + m(10\,cm)^2$

For radius of gyration

$I_{PQ} = mk^2$

$k^2 = \frac{2}{5}R^2 + (10\,cm)^2$

$= \frac{2}{5}(5)^2 + 100$

$= 10 + 100 = 110$

$k = \sqrt{110}\,cm$

$x = 110$

Question12

A uniform solid cylinder with radius R and length L has moment of inertia I_1, about the axis of cylinder. A concentric solid cylinder of

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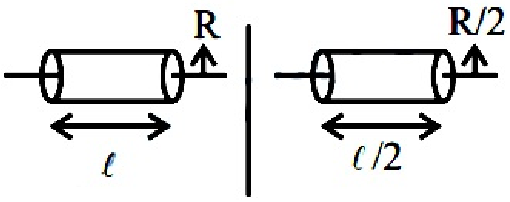
radius $R' = \frac{R}{2}$ and length $L' = \frac{L}{2}$ is caned out of the original cylinder. If I_2 is the moment of inertia of the carved out portion ot the cylinder then $\frac{I_1}{I_2}$ = ___

[24-Jan-2023 Shift 2]

Answer: 32

Solution:

Solution:



$I_1 = \frac{m_1 R^2}{2}$ $I_2 = \frac{m_2 (R/2)^2}{2}$

$\frac{I_1}{I_2} = \frac{4m_1}{m_2} = \frac{4 \cdot \rho \pi R^2 \ell}{\rho \cdot \frac{\pi R^2}{4} \times \frac{\ell}{2}} \Rightarrow \frac{I_1}{I_2} = 32$

Question13

A car is moving with a constant speed of 20 m / s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take $g = 10\ m/s^2$)

[25-Jan-2023 Shift 1]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C

Solution:

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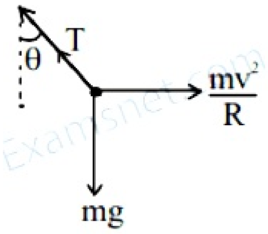


$T\cos\theta = mg$

$T\sin\theta = \frac{mv^2}{R}$

$\tan\theta = \frac{v^2}{Rg}$

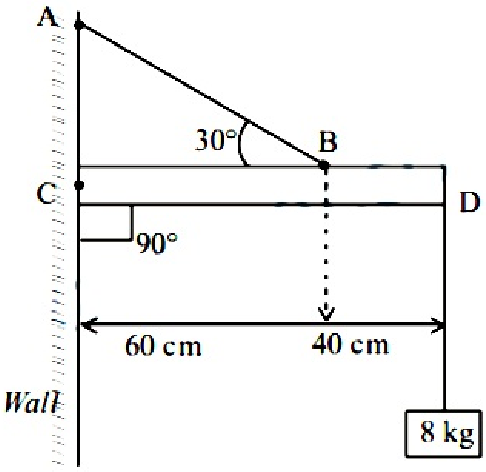
$\tan\theta = \frac{20^2}{40 \times 10}$

$\tan\theta = 1$

$\Rightarrow \theta = \frac{\pi}{4}$

Question14

An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is :
(Take $g = 10m/s^2$)



[25-Jan-2023 Shift 1]

Options:

A. 240N

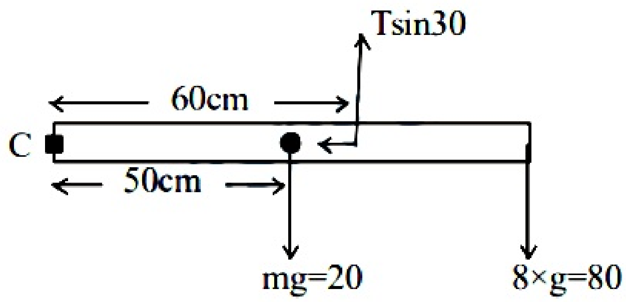
B. 90N

C. 300N

D. 30N

Answer: C

Solution:



Taking torque about point C

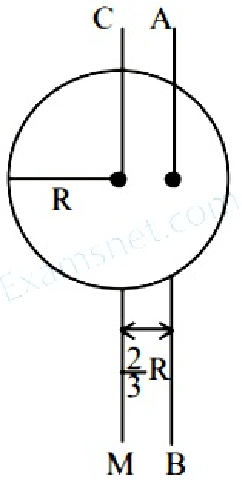
$\frac{T}{2} \times 60 = 20 \times 50 + 80 \times 100$

$\Rightarrow 3T = 100 + 800$

$\Rightarrow = 300N$

Question15

I_{CM} is moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. I_{AB} is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance 2 / 3R from center. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x : 9. The value of x is ________.



[25-Jan-2023 Shift 1]

Solution:

$$I_{cm} = \frac{mR^2}{2}$$

$$I_{AB} = \frac{mR^2}{2} + m\left(\frac{2R}{3}\right)^2 = \frac{17}{18}mR^2$$

$$\frac{I_{AB}}{I_{cm}} = \frac{17}{9} \Rightarrow x = 17$$

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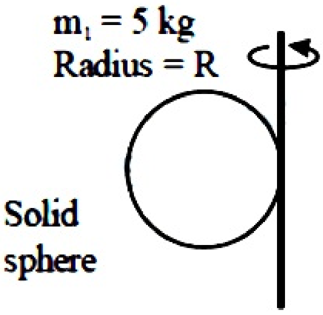
Question16

If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. The value of x is ______.
[25-Jan-2023 Shift 2]

Solution:

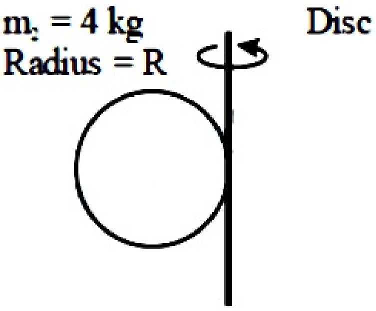
Solution:



$I_1 = \frac{2}{5}m_1R^2 + m_1R^2$

$I_1 = m_1R^2\left(\frac{7}{5}\right)$

$I_1 = 7R^2$



$I_2 = \frac{m_2R^2}{4} + m_2R^2$

$I_2 = \frac{5}{4}m_2R^2$

$I_2 = 5R^2$

$\frac{I_2}{I_1} = \frac{5}{7}$

$x = 5$

Question17

A car is moving on a horizontal curved road with radius 50m. The approximate maximum speed of car will be, if friction between tyres and road is 0.34. [. Take $g = 10ms^{-2}$]

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[29-Jan-2023 Shift 1]

Options:

A. $3.4ms^{-1}$

B. $22.4ms^{-1}$

C. $13ms^{-1}$

D. $17ms^{-1}$

Answer: C

Solution:

Solution:

$f_s = \frac{mv^2}{r}$

For maximum speed in safe turning,

$f_5 = f_5 \max = \mu mg$

v_{max} (for safe turning $= \sqrt{\mu\mu g}$

$= \sqrt{0.34 \times 50 \times 10} \approx 13m / s$

Question18

A solid sphere of mass 2 kg is making pure rolling on a horizontal surface with kinetic energy 2240J. The velocity of centre of mass of the sphere will be _______ ms^{-1}.
[29-Jan-2023 Shift 1]

Solution:

$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$2240 = \frac{1}{2}2(v)^2 + \frac{1}{2}\frac{2}{5}(2)R^2 \cdot \left(\frac{v}{R}\right)^2$

$2240 = v^2 + \frac{2}{5}v^2$

$\Rightarrow v = 40m / s$

Question19

An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at $x = +2m$, its velocity is $-4\hat{j}\,m / s$. The object's velocity (v) and

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acceleration (a) at x = −2m will be
[29-Jan-2023 Shift 2]

Options:

A. $v = 4\hat{i} m/s, a = 8\hat{j} m/s^2$

B. $v = 4\hat{j} m/s, a = 8\hat{i} m/s^2$

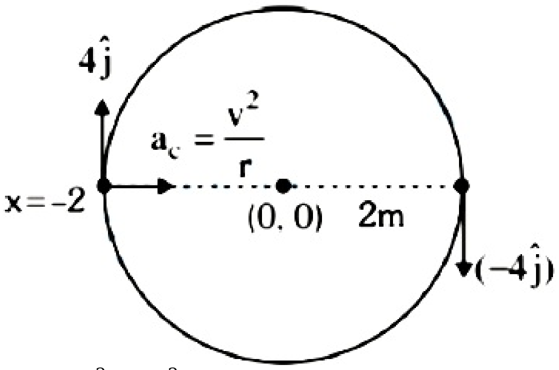
C. $v = -4\hat{j} m/s, a = 8\hat{i} m/s^2$

D. $v = -4\hat{i} m/s, a = -8\hat{j} m/s^2$

Answer: B

Solution:

Solution:



$a_c = \frac{V^2}{r} = \frac{4^2}{2} = \frac{16}{2} = 8m/s^2$

$\vec{V} = 4\hat{j}$

$a_{\vec{c}} = 8\hat{i}$

Question20

A car is moving on a circular path of radius 600m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km / hr is $t(1 - e^{-\pi/2})$ s. The value of t is _______ .
[29-Jan-2023 Shift 2]

Answer: 40

Solution:

Solution:

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$v\frac{dv}{dx} = \frac{v^2}{R} \Rightarrow \int_{15}^{v}\frac{dv}{v} = \frac{1}{R}\int_0^x dx$

$v = 15e^{x/R}$

$\frac{dx}{dt} = 15e^{x/R}$

$\int_0^{\frac{\pi R}{2}} e^{-x/R}\,dx = 15\int_0^{t_0} dt$

$t_0 = 40(1 - e^{-\pi/2})$

Question21

A thin uniform rod of length 2m. cross sectional area ' A ' and density ' d ' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If value of ω in terms of its rotational kinetic energy E is $\sqrt{\frac{\alpha E}{Ad}}$ then the value of α is _______.

[30-Jan-2023 Shift 1]

Answer: 3

Solution:

Solution:

$(KE)_{Rotational} = \frac{1}{2}I\omega^2 = E$

$E = \frac{1}{2}\frac{m\ell^2}{12}\omega^2$

$E = \frac{1}{2}\frac{dA\ell^3}{12}\omega^2$

$E = \frac{dA(2)^3}{24}\omega^2$

$\sqrt{\frac{3E}{dA}} = \omega$

$\alpha = 3$ Ans.

Question22

A uniform disc of mass 0.5 kg and radius r is projected with velocity 18m / s at t = 0 s on a rough horizontal surface. It starts off with a purely sliding motion at t = 0 s. After 2 s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2 s will be _______ J.

(given, coefficient of friction is 0.3 and g = 10m / s^2)

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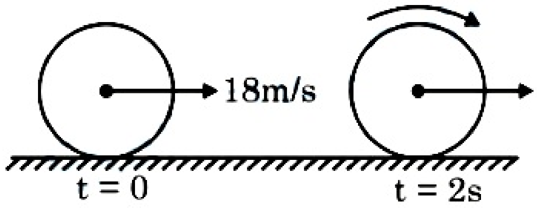

[30-Jan-2023 Shift 2]

Solution:

$a = -\mu_k g = -3$

$V = 18 - 3 \times 2$

$V = 12 m / s$

$KE = \frac{1}{2} mv^2 + \frac{1}{2} \frac{mr^2}{2} \frac{v^2}{r^2}$

$KE = \frac{3}{4} mv^2$

$KE = 3 \times 18 = 54 J$

Question23

Two discs of same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3:5. The moment of inertia of these discs respectively about their diameters will be in the ratio of −. The value of x is _______.
[31-Jan-2023 Shift 2]

Answer: 5

Solution:

Solution:

$m = \rho \pi R^2 t$

so $R^2 = \frac{m}{\rho \pi t}$

$I = \frac{mR^2}{4} = \frac{m^2}{4\rho \pi t}$

So $\frac{I_1}{I_2} = \frac{\rho_2 t_2}{\rho_1 t_1} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$

So $x = 5$

Question24

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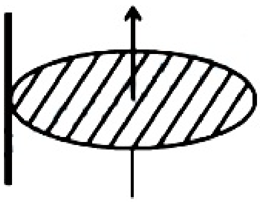
Moment of inertia of a dise of mass M and radius ' R ' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $\frac{x}{2}MR^2$. The value of x is ______.
[1-Feb-2023 Shift 2]

Answer: 3

Solution:

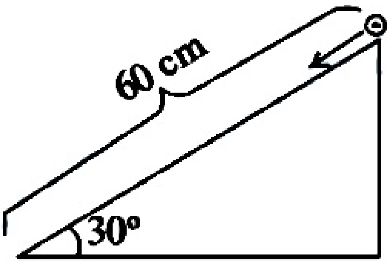
Solution:



$I = I_{cm} + Md^2$
$= \frac{MR^2}{2} + MR^2$
$= \frac{3}{2}MR^2$
$x = 3$

Question25

A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is ________ ms^{-1}. (Given $g = 10ms^{-2}$)



[1-Feb-2023 Shift 1]

Answer: 2

Solution:

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$$v = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$$

Where $h = 60 \sin 30^{\circ} = 30\,cm$

$k^2 = \frac{R^2}{2}$

$v = 2ms^{-1}$

Question26

Two identical solid spheres each of mass 2 kg and radii 10 cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40 cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is ______ $\times 10^{-3}\,kg - m^2$
[6-Apr-2023 shift 1]

Answer: 176

Solution:

Solution:
Using parallel axix theorem,

$$I_{sys} = \left(\frac{2}{5}mr^2 + md^2\right) \times 2$$

$$\Rightarrow I_{sys} = \left(\frac{2}{5} \times 2 \times 0.01 + 2 \times 0.04\right) \times 2 = (0.008 + 0.08) \times 2 = 0.088 \times 2 = 176 \times 10^{-3}$$

Question27

A ring and a solid sphere rotating about an axis passing trough their centers have same radii of gyration. The axis of rotation is perpendicular to plane of ring. The ratio of radius of ring to that of sphere is $\sqrt{\frac{2}{x}}$. The value of x is ______.
[6-Apr-2023 shift 2]

Solution:

Solution:
Given radius of gyration is same for ring and solid sphere

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$K_R = K_{ss}$

$R_R = \sqrt{\frac{2}{5}} R_{ss}$

or $\frac{R_R}{R_{ss}} = \sqrt{\frac{2}{5}}$

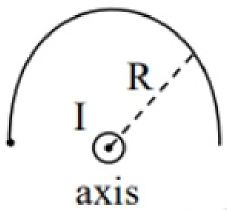
therefore $x = 5$

Question28

The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x} M R^2$, where R is the radius and M is the mass of the semicircular ring. The value of x will be _______.
[8-Apr-2023 shift 1]

Solution:

Solution:



$I = \int d\, mR^2 \Rightarrow R^2 \int dm = MR^2$

$I = MR^2$

Given $I = \frac{1}{x} M M^2$

$x = 1$

Question29

A force of $-P\hat{k}$ acts on the origin of the coordinate system. The torque about the point $(2, -3)$ is $P\left(\hat{a}\hat{i} + b\hat{j}\right)$, The ratio of $\frac{a}{b}$ is $\frac{x}{2}$. The value of x is _______.
[10-Apr-2023 shift 2]

Answer: 3

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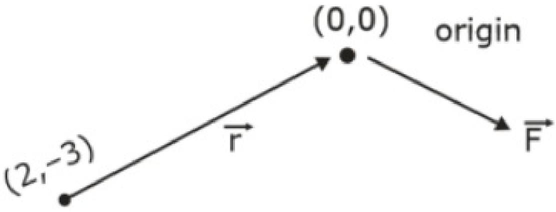
$\vec{\tau} = \vec{r} \times \vec{F}$



$\vec{\tau} = \text{head} - \text{tail}$

$= (0 - 2)\hat{i} + (0(-3))\hat{j}$

$= -2\hat{i} + 3\hat{j}$

$\tau = (-2\hat{i} + 3\hat{j}) \times (-p\hat{k})$

$= -2\hat{p} - 3p\hat{i}$

$= -p(3p\hat{i} + 2\hat{j})\ \frac{a}{b} = \frac{3}{2} = \frac{x}{2}\ x = 3$

Question30

A solid sphere of mass 500g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1}. If the moment of inertia of the sphere about its tangent is $x \times 10^{-2}$ times its angular momentum about the diameter. Then the value of x will be ________.
[11-Apr-2023 shift 1]

Solution:

$I_1 = \frac{2}{5}mR^2$

$I_2 = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$

oment about diameter is

$L_{com} = I_1 w = \frac{2}{5}mR^2 w$

Now,

$\frac{I_2}{L_{com}} = \frac{\frac{7}{5}mR^2}{\frac{2}{5}mR^2 w} = \frac{7}{2w}$

$\frac{I_2}{L_{com}} = \frac{7}{2 \times 10} = \frac{7}{20}$

Now $\frac{7}{20} = x \times 10^{-2}$

$x = \frac{7}{20} \times 100$

$x = 35$

Ans.

Question31

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A circular plate is rotating horizontal plane, about an axis passing through its center perpendicular to the plate, with an angular velocity ω. A person sits at the center having two dumbbells in his hands. When he stretches out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$. The value of x is ______
[11-Apr-2023 shift 2]

Answer: 3

Solution:

Solution:

$I_1\omega_1 = I_2\omega_2$

$I\omega = 3I\omega_2$

$\omega_2 = \frac{\omega}{3}$

$E = \frac{1}{2}I\omega^2$

$E_f = \frac{1}{2} \times 3I \times \left(\frac{\omega}{3}\right)^2 = \frac{\frac{1}{2}I\omega^2}{3}$

$x = 3$

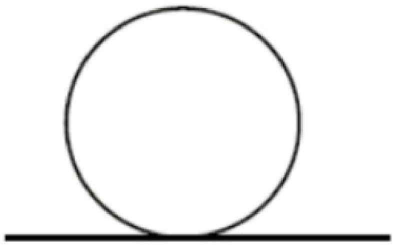
Question32

For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is $\frac{x}{5}$. The value of x is ______.
[12-Apr-2023 shift 1]

Answer: 2

Solution:

Solution:



$KE_T \frac{1}{2}mV^2$

$KE_R = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{3}mR^2 \cdot \frac{V^2}{R^2} = \frac{1}{3}mV^2$

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$$KE_T = \frac{1}{2}mV^2 + \frac{mV^2}{3}$$
$$KE_T = \frac{5}{6}mV^2$$
$$\frac{K_R}{K_T} = \frac{1/3mV^2}{5/6mV^2} = \frac{2}{5}$$
$x = 2$

Question33

A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is π : 22 the, the value of its angular speed will be rad / s.
[13-Apr-2023 shift 1]

Answer: 4

Solution:

Solution:
Angular momentum,
$$L = I\omega$$
$$L = \frac{2}{5}MR^2\omega$$
$$\text{Energy} = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$
$$E = \frac{1}{2}M(\omega R)^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$
$$= \frac{7}{10}M\omega^2R^2$$
$$\frac{L}{E} = \frac{4}{7\omega} = \frac{\pi}{22}$$
$$\omega = \frac{88}{7\pi} = \frac{88}{7\times\frac{22}{7}} = 4\text{ rad / s}$$

Question34

A light rope is wound around a hollow cylinder of mass 5 kg and radius 70 cm. The rope is pulled with a force of 52.5N. The angular acceleration of the cylinder will be _______ rads^{-2}.
[13-Apr-2023 shift 2]

Solution:

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$\tau = I\alpha$
$FR = mR^2\alpha$
$\propto = \frac{F}{mR} = \frac{52.5}{5 \times 0.7} = 15 rads^{-2}$

Question35

A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyrations respectively ($k_{sph} : k_{eyl}$) is $2 : \sqrt{x}$. The value of x is ______.
[15-Apr-2023 shift 1]

Solution:

$I_{sphere} = \frac{2}{5}MR^2 = Mk^2$

$k_{sph} = \sqrt{\frac{2}{5}}R$

$I_{cylinder} = \frac{MR^2}{2} = Mk^2$

$k_{cyl} = \frac{R}{\sqrt{2}}$

$\frac{k_{sph}}{k_{cyl}} = \frac{2}{\sqrt{5}}$

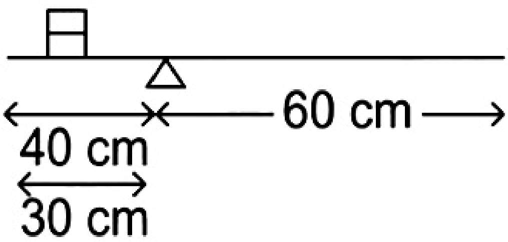
$x = 5$

Question36

A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be $x \times 10^{-2}$ kg. The value of x is__
[24-Jun-2022-Shift-1]

Answer: 6

Solution:



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If λ is the mass per unit length of the scale then
$0.02 \times (30) \times 10 + \lambda 40 \times 20 \times 10 = \lambda 60 \times 30 \times 10$
$0.006 = \lambda 10$
or $100\lambda = 0.06\,\text{kg}$
$= 6 \times 10^{-2}\,\text{kg}$
$\Rightarrow x = 6$

Question37

A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be:
[24-Jun-2022-Shift-2]

Options:

A. 7.5 rad

B. 15 rad

C. 20 rad

D. 30 rad

Answer: B

Solution:

Solution:
$\theta_1 = \frac{1}{2}\alpha(2 \times 1 - 1) = 5\,\text{rad}$
$\Rightarrow \alpha = 10\,\text{rad / sec}^2$
So $\theta_2 = \frac{1}{2} \times \alpha(2 \times 2 - 1) = 15\,\text{rad}$

Question38

If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :
[25-Jun-2022-Shift-1]

Options:

A. $3\hat{i} + 4\hat{j} - 2\hat{k}$

B. $-10\hat{i} + 10\hat{j} + 5\hat{k}$

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C. $10\hat{i} + 5\hat{j} - 10\hat{k}$

D. $10\hat{i} + \hat{j} - 5\hat{k}$

Answer: B

Solution:

Solution:

$\vec{\tau} = \vec{r} \times \vec{F}$

$= (2\hat{i} + \hat{j} + 2\hat{k}) \times (3\hat{i} + 4\hat{j} - 2\hat{k})$

$= -10\hat{i} + 10\hat{j} + 5\hat{k}$

Question39

Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '$2R'$ are as follows:
$I_1 =$ M.I. of solid sphere about its diameter
$I_2 =$ M.I. of solid cylinder about its axis
$I_3 =$ M.I. of solid circular disc about its diameter
$I_4 =$ M.I. of thin circular ring about its diameter
If $2(I_2 + I_3) + I_4 = x \cdot I_1$, then the value of x will be
[25-Jun-2022-Shift-2]

Solution:

Solution:

$2\left(\frac{1}{2} + \frac{1}{4}\right) \times M(2R)^2 + \frac{1}{2}M(2R)^2 = x\frac{2}{5}M(2R)^2$

$\Rightarrow 1 + \frac{1}{2} + \frac{1}{2} = x \times \frac{2}{5}$

$\Rightarrow x = 5$

Question40

A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 $rads^{-1}$ in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in $rads^{-1}$).
[26-Jun-2022-Shift-1]

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Options:

A. $\frac{M}{(M + m)}$

B. $\frac{(M + 2m)}{2M}$

C. $\frac{2M}{(M + 2m)}$

D. $\frac{2(M + 2m)}{M}$

Answer: C

Solution:

Solution:

$I_1\omega_1 = I_2\omega_2$

$MR^2\omega_1 = (MR^2 + 2mR^2)\omega_2$

$\omega_2 = \left(\frac{M}{M + 2m}\right)\omega_1$

$\omega_2 = 2\left(\frac{M}{M + 2m}\right)$

Question41

A solid spherical ball is rolling on a frictionless horizontal plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is [26-Jun-2022-Shift-2]

Options:

A. $\frac{2}{5}$

B. $\frac{2}{7}$

C. $\frac{1}{5}$

D. $\frac{7}{10}$

Answer: B

Solution:

Solution:

$KE_R = \frac{1}{2}I w^2$

$= \frac{1}{2} \times \frac{2}{5} \times \omega^2 \times (mR^2)$

$KE_{total} = \frac{1}{2} \times \frac{7}{5} \times mR^2 \times \omega^2$

$\therefore \frac{KE_R}{KE_{total}} = \frac{2}{7}$

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Question42

Choose the correct answer from the options given below :

	List-I		List-II
(A)	Moment of inertia of solid sphere of radius R about any tangent.	(I)	$\frac{5}{3}MR^2$
(B)	Moment of inertia of hollow sphere of radius (R) about any tangent.	(II)	$\frac{7}{5}MR^2$
(C)	Moment of inertia of circular ring of radius (R) about its diameter.	(III)	$\frac{1}{4}MR^2$
(D)	Moment of inertia of circular disc of radius (R) about any diameter.	(IV)	$\frac{1}{2}MR^2$

[28-Jun-2022-Shift-1]

Options:

A. A - II, B - I, C - IV, D - III

B. A - I, B - II, C - IV, D - III

C. A - II, B - I, C - III, D - IV

D. A - I, B - II, C - III, D - IV

Answer: A

Solution:

Solution:

(A) Moment of inertia of solid sphere of radius R about a tangent $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2 \Rightarrow A - (II)$

(B) Moment of inertia of hollow sphere of radius R about a tangent $= \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2 \Rightarrow B - (I)$

(C) Moment of inertia of circular ring of radius (R) about its diameter $= \frac{(MR^2)}{2} \Rightarrow C - (IV)$

(D) Moment of inertia of circular disc of radius (R) about any diameter

$= \frac{MR^2/2}{2} = \frac{MR^2}{4}$

$\Rightarrow D - (III)$

Question43

A $\sqrt{34}$m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3m away from the wall as shown in the figure. If E_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w / F_f

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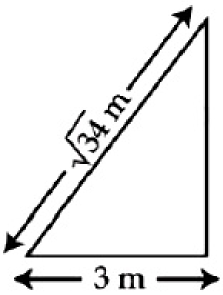
will be :
(Use $g = 10 m / s^2$.)



[28-Jun-2022-Shift-2]

Options:

A. $\frac{6}{\sqrt{110}}$

B. $\frac{3}{\sqrt{113}}$

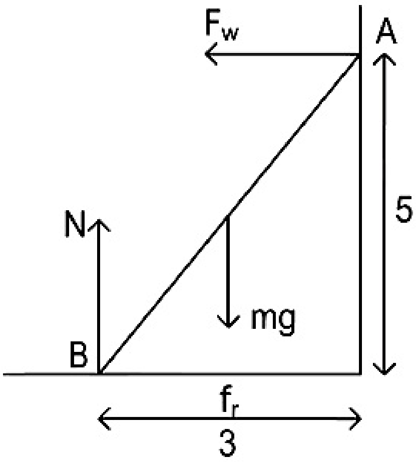
C. $\frac{3}{\sqrt{109}}$

D. $\frac{2}{\sqrt{109}}$

Answer: C

Solution:

Solution:



Taking torque from B

$F_w \times 5 = \frac{3}{2}mg$

$\Rightarrow F_w = \frac{3}{10} \times 10 \times 10$

$= 30N$

$N = mg = 100N$

and $f_r = F_w = 30N$

so $F_f = \sqrt{N^2 + f_r^2} = \sqrt{10900} = 10\sqrt{109}N$

so $\frac{F_w}{F_f} = \frac{3}{\sqrt{109}}$

Question44

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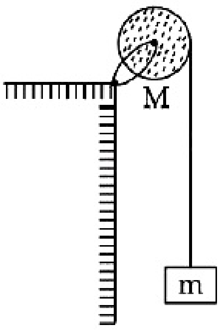
A uniform disc with mass M = 4 kg and radius R = 10 cm is mounted on a fixed horizontal axle as shown in figure. A block with mass m = 2 kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is ____N.
(. Take .$g = 10 ms^{-2}$)

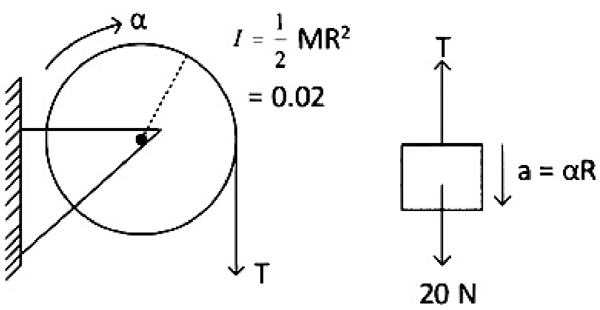


[28-Jun-2022-Shift-2]

Answer: 10

Solution:

Solution:



$20 - T = 2a$

and $0.1 \times T = 0.02\alpha = \frac{0.02a}{0.1}$

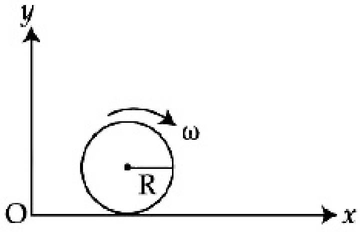
$T = 2a$

$\Rightarrow a = 5 m / sec^2$

So $T = 10N$

Question45

A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin 0 is $\frac{a}{3}R^2\omega$. The value of a will be:



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[29-Jun-2022-Shift-1]

Options:

A. 2

B. 3

C. 5

D. 4

Answer: C

Solution:

Solution:
$L_0 =$ angular momentum of shell about O.
As shell is rolling
so $V_{cm} = \omega R$
$L_0 = mV_{cm}R + I\omega$
$= 1 \times \omega R \times R + \frac{2}{3}R^2\omega$
$= \frac{5}{3}R^2\omega$
so a = 5

Question46

The moment of inertia of a uniform thin rod about a perpendicular axis passing through one end is I_1. The same rod is bent into a ring and its moment of inertia about a diameter is I_2. If $\frac{I_1}{I_2}$ is $\frac{x\pi^2}{3}$, then the value of x will be___
[29-Jun-2022-Shift-2]

Solution:

$$I_1 = \frac{ML^2}{3} \ldots..(1)$$

$$I_2 = \frac{MR^2}{2} \text{ and } 2\pi R = L$$

$$\Rightarrow I_2 = \frac{M}{2}\left(\frac{L^2}{4\pi^2}\right) \ldots..$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{8\pi^2}{3}$$

$$\Rightarrow x = 8$$

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Question47

A solid cylinder and a solid sphere, having same mass M and radius R_v roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be :
[25-Jul-2022-Shift-1]

Options:

A. $\sqrt{\frac{5}{3}}$

B. $\sqrt{\frac{4}{5}}$

C. $\sqrt{\frac{3}{5}}$

D. $\sqrt{\frac{14}{15}}$

Answer: D

Solution:

Solution:

$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

$v = \sqrt{\frac{2Sg\sin\theta}{1+\frac{K^2}{R^2}}}$

$\Rightarrow \frac{v_c}{v_{ss}} \sqrt{\frac{1+\frac{K_{ss}^2}{R^2}}{1+\frac{K_c^2}{R^2}}} = \sqrt{\frac{1+\frac{2}{5}}{1+\frac{1}{2}}}$

$\Rightarrow \sqrt{\frac{\frac{7}{5}}{\frac{3}{2}}} = \sqrt{\frac{14}{15}}$

Question48

A disc of mass 1 kg and radius R is free to rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}\,\text{rad s}^{-1}$ where x = ______.

$(g = 10\text{ms}^{-2})$
[26-Jul-2022-Shift-1]

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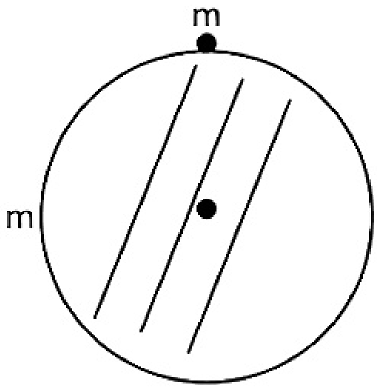
Answer: 5

Solution:

Solution:



Loss in P.E. = Gain in K.E.

$$2mgR = \frac{1}{2}\left[\frac{1}{2}mR^2 + mR^2\right]w^2$$

$$2mgR = \frac{1}{2} \times \frac{3}{2}mR^2w^2$$

$$w^2 = \frac{8g}{3R}$$

$$w = \sqrt{\frac{8g}{3R}} = 4\sqrt{\frac{g}{2 \times 3R}}$$

$$\Rightarrow x = \frac{g}{2} = 5$$

Question49

The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be ______m.
Given, the length of the rod is $10\sqrt{3}$m.
[26-Jul-2022-Shift-2]

Solution:

Question50

A pulley of radius 1.5m is rotated about its axis by a force

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$F = (12t - 3t^2)N$ applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is $4.5\,kg\,m$, the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is ________.
[27-Jul-2022-Shift-1]

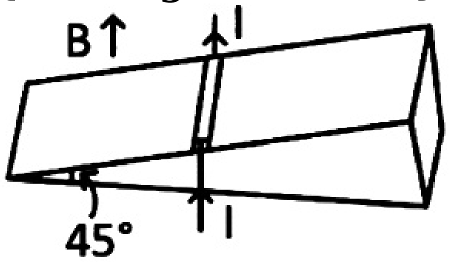
Answer: 18

Solution:

Solution:
$\tau = I\alpha \Rightarrow (12t - 3t^2)1.5 = 4.5\alpha$
$\Rightarrow \alpha = 4t - t^2$
$\Rightarrow \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = \int_0^t (4t - t^2)\,dt$
$\Rightarrow \omega = 2t^2 - t^3 / 3$

Question51

As shown in the figure, a metallic rod of linear density $0.45\,kg\,m^{-1}$ is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The minimum current flowing in the rod required to keep it stationary, when 0.15T magnetic field is acting on it in the vertical upward direction, will be :
{ . Use .g = $10m / s^2$ }



[28-Jul-2022-Shift-1]

Options:

A. 30A

B. 15A

C. 10A

D. 3A

Answer: A

Solution:

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$mg \times \frac{1}{\sqrt{2}} = \frac{il\,B}{\sqrt{2}}$

$\Rightarrow i = \frac{mg}{Bl}$

$= \frac{0.45 \times 10}{0.15} = 30A$

Question52

The torque of a force $5\hat{i} + 3\hat{j} - 7\hat{k}$ about the origin is τ. If the force acts on a particle whose position vector is 2i + 2j + k, then the value of τ will be
[29-Jul-2022-Shift-2]

Options:

A. $11\hat{i} + 19\hat{j} - 4\hat{k}$

B. $-11\hat{i} + 9\hat{j} - 16\hat{k}$

C. $-17\hat{i} + 19\hat{j} - 4\hat{k}$

D. $17\hat{i} + 9\hat{j} + 16\hat{k}$

Answer: C

Solution:

Solution:

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 5 & 3 & -7 \end{vmatrix}$$

$= \hat{i}(-14 - 3) + \hat{j}(5 + 14) + \hat{k}(6 - 10)$

$= -17\hat{i} + 19\hat{j} - 4\hat{k}$

Question53

Four identical solid spheres each of mass m and radius a are placed with their centres on the four corners of a square of side b. The moment of inertia of the system about one side of square, where the axis of rotation is parallel to the plane of the square is
[26 Feb 2021 Shift 1]

Options:

A. $\frac{4}{5}ma^2 + 2mb^2$

B. $\frac{8}{5}ma^2 + mb^2$

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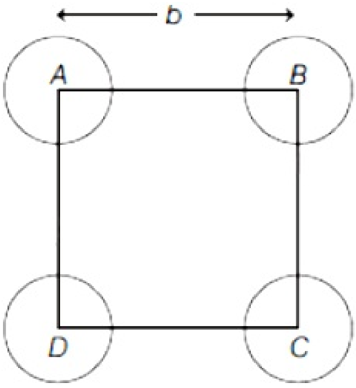
C. $\frac{8}{5}ma^2 + 2mb^2$

D. $\frac{4}{5}ma^2$

Answer: C

Solution:

Solution:
Given, mass of solid sphere $= m$
Radius of solid sphere $= a$
Side of square $= b$
Let I_{net} be the net moment of inertia of system.
Moment of inertia of sphere, $I_s = \frac{2}{5}ma^2$
Axis of rotation is BC.



∴ Moment of inertia of any body at distance,
$(\because d = b)$
$d = md^2 = mb^2$
$\therefore I_{net} = 4\left(\frac{2}{5}ma^2\right) + 2(mb^2)$
$\therefore {}_{net} = 4\left(\frac{2}{5}ma^2\right) + 2(mb^2)$
$\Rightarrow I_{net} = \frac{8}{5}ma^2 + 2mb^2$

Question54

A uniform thin bar of mass 6kg and length 2.4m is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is .... $\times 10^{-1}$kg − m^2.
[24 Feb 2021 Shift 2]

Answer: 8

Solution:

Solution:
Given, mass of uniform bar, $m = 6kg$
Length of bar, $I = 2.4m$

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Side of hexagon, $a = \frac{2.4}{6} = 0.4m$
Mass of each side, $m' = 6/6 = 1kg$
and $OP = \sqrt{a^2 - a^2/4} = a\sqrt{3}/2$
$= 0.2\sqrt{3}m$
Now, by using parallel axes theorem,

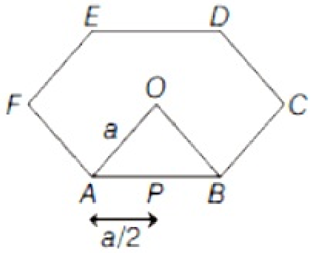
$$I_{OP} = \frac{m'a^2}{12} + m'\left(\frac{\sqrt{3}a}{2}\right)^2$$

$$= \frac{a^2}{12} + \frac{3a^2}{4} = \frac{10a^2}{12} = \frac{5a^2}{6}$$

$$\Rightarrow I_{OP} = \frac{5}{6} \times 0.4 \times 0.4$$

and I_{net} (net moment of inertia at O)

$$= 6 \times \frac{5}{6} \times 0.4 \times 0.4 = 8 \times 10^{-1} kg - m^2$$



Question55

Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration a, then the acceleration of mass B will be

[26 Feb 2021 Shift 2]

Options:

A. $\frac{Ma - F}{M}$

B. $\frac{MF}{F + Ma}$

C. $\frac{F + Ma}{M}$

D. $\frac{F - Ma}{M}$

Answer: D

Solution:

Solution:
Let a_{CM} be the acceleration of centre of mass of system and F be the applied force on spring.

$$\therefore a_{CM} = \frac{F}{m_A + m_B} = \frac{m_A a_A + m_B a_B}{m_A + m_B}$$

$$\Rightarrow a_{CM} = \frac{F}{2M} = \frac{M a_A + M a_B}{2M} \quad (\because m_A = m_B = M)$$

$$\Rightarrow a_B = \frac{F - M a_A}{M} = \frac{F - M a}{M}$$

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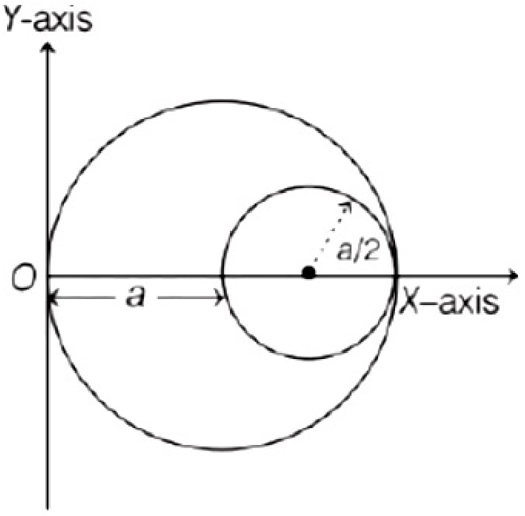
Question56

A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius a as shown in figure The centroid of the remaining circular portion with respect to point O will be



[24 Feb 2021 Shift 2]

Options:

A. $\frac{1}{6}a$

B. $\frac{10}{11}a$

C. $\frac{5}{6}a$

D. $\frac{2}{3}a$

Answer: C

Solution:

Solution:
Given, radius of hole, $r = a / 2$ and radius of disc, $R = a$
Let x_{CM} be the centre of mass of system,
m_1, x_1 be the mass and centre of mass of disc
m_2, x_2 be the mass and centre of mass of circular hole.

$\therefore m_2 = \frac{m_1}{\pi R^2}\pi r^2$

$\Rightarrow m_2 = \frac{m_1 r^2}{R^2} = \frac{m_1(a/2)^2}{a^2} = \frac{m_1}{4}$

and $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

$\therefore x_{CM} = \frac{m_1 a - (m_1/4)(3a/2)}{m_1 - (m_1/4)}$

$\Rightarrow x_{CM} = \frac{m_1 a(1 - 3/8)}{3m_1/4} \Rightarrow x_{CM} = \frac{5a}{6}$

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Question57

A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I . A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance h, the square of angular velocity of wheel will be
[26 Feb 2021 Shift 2]

Options:

A. $\frac{2mgh}{1 + 2mr^2}$

B. $\frac{2mgh}{1 + mr^2}$

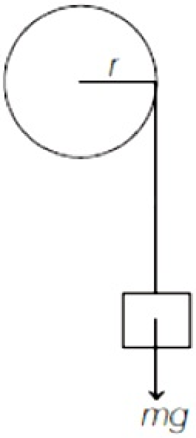
C. 2gh

D. $\frac{2gh}{1 + mr^2}$

Answer: B

Solution:

Solution:
Moment of inertia of wheel, $I = mR^2$



Let v be the velocity and $v = rw = c$
Let v be the velocity and $v = r\omega \Rightarrow \omega = v / r$
By using law of conservation of energy,
$1 / 2(I + mr^2)\omega^2 = mgh$
$\omega^2 = \frac{2mgh}{I + mr^2}$

Question58

A sphere of radius a and mass m rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upwards. Assuming that it rolls without slipping, how far up the sphere will travel?

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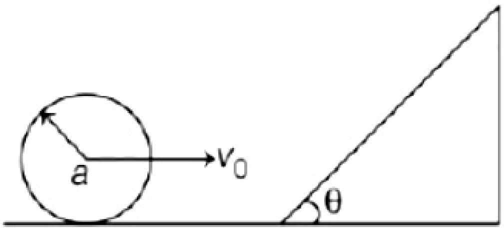


[25 Feb 2021 Shift 2]

Options:

A. $\frac{7v_0^2}{10g\sin\theta}$

B. $\frac{v_0^2}{5g\sin\theta}$

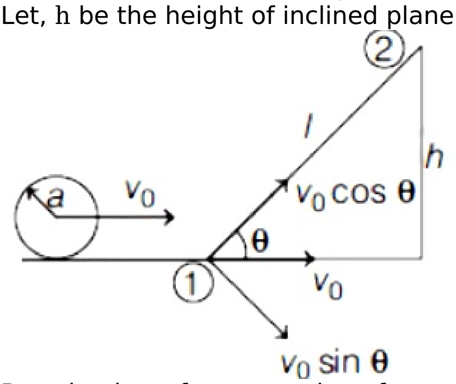
C. $\frac{10v_0^2}{7g\sin\theta}$

D. $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$

Answer: A

Solution:

Solution:
Given, radius of sphere is a, mass of sphere is m, horizontal speed of sphere is v_0.
Moment of inertia of solid sphere, $I = 2/5ma^2$
Let, h be the height of inclined plane and l be the length of inclined plane.



By using law of conservation of energy,
Total energy at 1 = Total energy at 2
$\therefore PE_1 + KE_1 = PE_2 + KE_2$
$\Rightarrow 0 + 1/2mv_0^2 + 1/2/\omega^2 = mgh...$ (i)

As, $v_0 = a\omega \Rightarrow \omega = \frac{v_0}{a}$

$\therefore \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\frac{v_0^2}{a^2} = mgh$

$\Rightarrow \frac{1}{2}mv_0^2(1 + 2/5) = mgh$

$\Rightarrow \frac{7v_0^2}{10} = gh ...$ (ii)

As, $\sin\theta = h/l$
$\Rightarrow h = I\sin\theta$
Put this value in Eq. (ii), we get

$\Rightarrow \frac{7}{10}v_0^2 = gl\sin\theta \Rightarrow l = \frac{7v_0^2}{10g\sin\theta}$

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Question59

Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as;
I_1 = M.I. of thin circular ring about its diameter,
I_2 = M.I. of circular disc about an axis perpendicular to the disc and going through the centre,
I_3 = M.I. of solid cylinder about its axis and I_4 = M.I. of solid sphere about its diameter. Then :
[24feb2021shift1]

Options:

A. $I_1 + I_3 < I_2 + I_4$

B. $I_1 + I_2 = I_3 + \frac{5}{2} I_4$

C. $I_1 = I_2 = I_3 > I_4$

D. $I_1 = I_2 = I_3 < I_4$

Answer: C

Solution:

Solution:

Moment of inertia of ring about its diameter, $I_1 = \frac{MR^2}{2}$

Moment of inertia of disc, $I_2 = \frac{MR^2}{2}$

Moment of inertia of solid cylinder, $I_3 = \frac{MR^2}{2}$

Moment of inertia of solid sphere, $I_4 = \frac{2}{5}MR^2 \therefore I_1 = I_2 = I_3 > I_4$

Question60

A thin circular ring of mass M and radius r is rotating about its axis with an angular speed . Two particles having mass m each are now attached at diametrically opposite points.
The angular speed of the ring will become
[18 Mar 2021 Shift 1]

Options:

A. $\omega \frac{M}{M + m}$

B. $\omega \frac{M + 2m}{M}$

C. $\omega \frac{M}{M + 2m}$

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D. $\omega \frac{M - 2m}{M + 2m}$

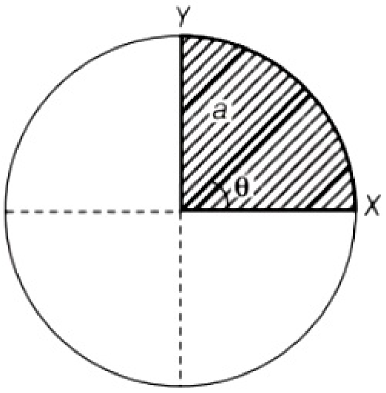
Answer: C

Solution:

Solution:
Angular momentum, $L = I\omega$
The moment of inertia of the circular ring, $I = Mr^2$
When masses are attached at diametrically opposite ends, the moment of inertia, $I' = Mr^2 + 2mr^2$
The new angular speed $= \omega'$
So, the new angular momentum, $L' = I'\omega'$
Using the law of conservation of angular momentum
$I\omega = I'\omega'$
$\Rightarrow Mr^2\omega = (Mr^2 + 2mr^2)\omega'$
$\Rightarrow \omega' = \frac{M}{M + 2m}\omega$

Question61

The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$, where x is ………… (Round off to the nearest integer) (a is an area as shown in the figure)



[17 Mar 2021 Shift 2]

Solution:

As we know the centre of mass of the quarter disc,
$= \left(\frac{4R}{3\pi}, \frac{4R}{3\pi}\right)$
Since $R = a$
∴ The centre of mass of the quarter disc is $\left(\frac{4a}{3\pi}, \frac{4a}{3\pi}\right)$.
Hence, the value of x is 4 .

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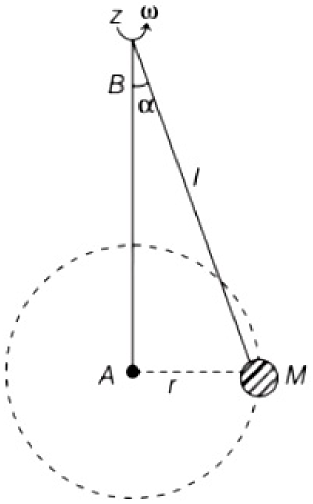
Question62

A mass M hangs on a massless rod of length / which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z-direction and the angular momentum of M about B is L_B. The correct statement for this system is



[17 Mar 2021 Shift 1]

Options:

A. L_A and L_B are both constant in magnitude and direction

B. L_B is constant in direction with varying magnitude

C. L_B is constant, both in magnitude and direction

D. L_A is constant, both in magnitude and direction

Answer: D

Solution:

Solution:
It is given that the system is assumed to be in steady circular motion with constant angular velocity ω.
We know that $L = m(r \times v)$
where,
$L =$ angular momentum,
$v =$ velocity of the particle,
$m =$ mass of the particle
and $r =$ radius of the circular path traced by the particle.
Since, L_A is the angular momentum of M about point A which lies in positive z-direction. Therefore, with respect to point A, we will get the direction of L along positive Z-axis and of constant magnitude of mvr.
Since, L_B is the angular momentum of M about point B, so with respect to point B, we will get the constant magnitude of L but its direction will be continuously changing. Hence, option (d) is correct.

Question63

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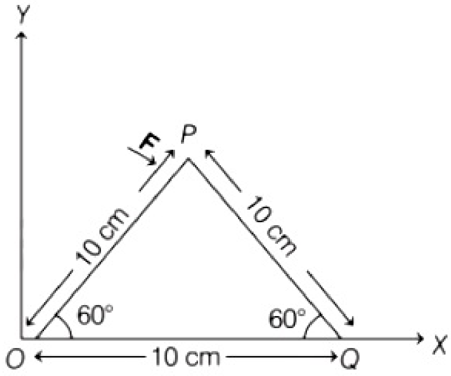
A triangular plate is shown below. A force $F = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point O and Q are



[17 Mar 2021 Shift 1]

Options:

A. $-15 - 20\sqrt{3}, 15 - 20\sqrt{3}$

B. $15 + 20\sqrt{3}, 15 - 20\sqrt{3}$

C. $15 - 20\sqrt{3}, 15 + 20\sqrt{3}$

D. $-15 + 20\sqrt{3}, 15 + 20\sqrt{3}$

Answer: A

Solution:

Solution:

Given, $F = 4\hat{i} - 3\hat{j}$

Resolving the components of position vector in horizontal and vertical direction.

∴ Horizontal component, $r_x = r\cos\theta$

$= 10\cos 60^\circ \quad \left[\because \cos 60^\circ = \frac{1}{2}\right]$

$= 10 \times \frac{1}{2} = 5$ units

Vertical component, $r_y = r\sin\theta$

$= 10\sin 60^\circ \quad \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$

$= 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ units

∴ Position vector w.r.t. O, $r_O = 5\hat{i} + 5\sqrt{3}\hat{j}$

and position vector w.r.t. Q,

$r_Q = -5\hat{i} + 5\sqrt{3}j$

Torque at point P w.r.t. O,

$\tau_O = r_O \times F = \left(5\hat{i} + 5\sqrt{3}\hat{j}\right) \times \left(4\hat{i} - 3\hat{j}\right)$

$= (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})\left(-\hat{k}\right)$

Torque at point P w.r.t. Q,

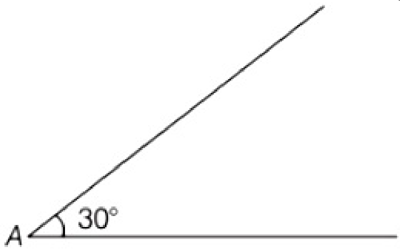
$\tau_Q = r_Q \times F = \left(4\hat{i} - 3\hat{j}\right) \times \left(-5\hat{i} + 5\sqrt{3}\hat{j}\right)$

$= (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})\left(-\hat{k}\right)$

Question64

A sphere of mass 2kg and radius 0.5m is rolling with an initial speed of

$1ms^{-1}$ goes up an inclined plane which makes an angle of 30° with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A ?



[17 Mar 2021 Shift 2]

Options:

A. 0.60s

B. 0.52s

C. 0.57s

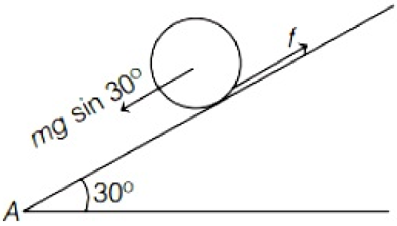
D. 0.80s

Answer: C

Solution:

Solution:

As we know the moment of inertia of the solid sphere, $I = 2/5mR^2$



Acceleration of sphere on inclined plane, $a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$

Substituting the values in the above equation, we get

$a = \frac{g\sin 30^{\circ}}{1 + \frac{\frac{2}{5}mR^2}{mR^2}} = \frac{10 + 1/2}{(1 + 2/5)} = \frac{5 \times 5}{7}$

$a = 3.5m/s^2$

At the top of incline,

$v = u + at \Rightarrow 0 = 1 - 3.5t$

$t = 1/3.5s$

∴ Total time = time of ascent + time of decent

As, time of ascent = time of decent

⇒ Total time = 2 (time of ascent)

Total time = $2(1/3.5)s$

Total time = $0.57s$

Hence, the total time taken to sphere to return the starting point A is 0.57s.

Question65

The following bodies,
1. a ring
2. a disc

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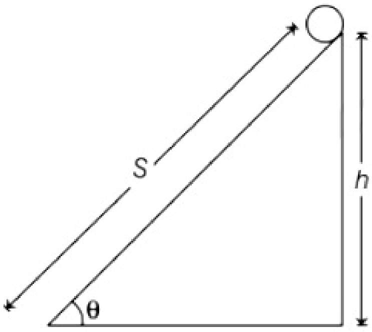
3. a solid cylinder
4. a solid sphere
of same mass m and radius R are allowed t roll down without slipping simultaneously from the top of the inclined plane. The boo which will reach first at the bottom of the inclined plane is ....
(Mark the body as per their respective numbering given in the question)



[17 Mar 2021 Shift 1]

Answer: 4

Solution:

Solution:
For rolling without slipping on an inclined plane, we can write
$Rmg\sin\theta = (mK^2 + mR^2)\alpha$
$\Rightarrow Rmg\sin\theta = m(K^2 + R^2)\alpha$
$\Rightarrow \alpha = \frac{Rg\sin\theta}{K^2 + R^2} \Rightarrow \frac{\alpha}{R} = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}}$
$\Rightarrow a = \frac{g\sin\theta}{1 + \frac{K^2}{R^2}} \quad \left[\because a = \frac{\alpha}{R}\right] \ldots$ (i)
Time period, $t = \sqrt{\frac{2s}{a}}$ ...(ii)
From Eqs. (i) and (ii), we get
$t = \sqrt{\frac{2s}{g\sin\theta}\left(1 + \frac{k^2}{R^2}\right)}$
For least time acceleration a should be maximum and K should be minimum and we know that K is least for solid sphere. So, time will be least for sphere.
It means the body which will reach first at the bottom of the inclined plane is 4 , i.e. solid sphere.

Question66

The angular speed of truck wheel is increased from 900 rpm to 2460rpm in 26s. The number of revolutions by the truck engine during this time is ....
(Assuming the acceleration to be uniform).
[17 Mar 2021 Shift 1]

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Answer: 728

Solution:

Solution:

Initial angular speed, $\omega_0 = 900\text{rpm} = 900 \times \frac{2\pi}{60} = 30\pi\text{rad/s}$

Final angular speed, $\omega = 2460\text{rpm}$

$= 2460 \times \frac{2\pi}{60} = 41 \times 2\pi = 82\pi\text{rad/s}$

We know that,

$\omega = \omega_0 + \alpha t \Rightarrow \omega - \omega_0 = \alpha t$

$\Rightarrow 82\pi - 30\pi = \alpha \times 26 \Rightarrow 52\pi = 26\alpha$

$\Rightarrow \alpha = 2\pi\text{rad/s}^2$

Also we know that,

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow 2\pi n = 30\pi \times 26 + \frac{1}{2} \times 2\pi \times (26)^2$

$\Rightarrow 2n = 30 \times 26 + (26)^2 \Rightarrow n = \frac{30 \times 26 + (26)^2}{2}$

$= \frac{26(30 + 26)}{2} = 13 \times 56 = 728$

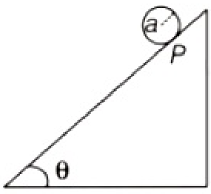
Question67

A solid disc of radius a and mass m rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$, where b is

(Round off to the nearest integer)

(g = acceleration due to gravity)

(theta = angle as shown in figure)



[16 Mar 2021 Shift 2]

Solution:

We know that, on an inclined plane,

Acceleration, $a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$

$\Rightarrow a = \frac{g\sin\theta}{1 + \frac{1}{2}}$ [∵ For disc, $I = \frac{mR^2}{2}$]

$\Rightarrow a = \frac{2}{3}g\sin\theta$.....(i)

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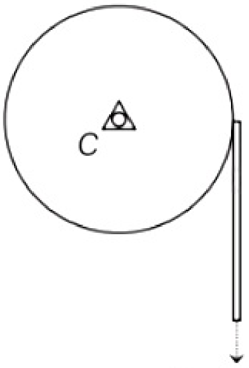
As per question, acceleration of the disc will be $\frac{2}{b} g \sin\theta$.
Comparing it with Eq. (i), we get
$b = 3$

Question68

Consider a 20kg uniform circular disc of radius 0.2m. It is pin supported at its centre and is at rest initially. The disc is acted upon by a constant force $F = 20N$ through a massless string wrapped around its periphery as shown in the figure.



Suppose the disc makes n number of revolutions to attain an angular speed of 50rad s^{-1}. The value of n to the nearest integer, is ....
(Given, in one complete revolution, the disc
[16 Mar 2021 Shift 1]

Solution:

Given, $M = 20\text{kg}$, $R = 0.2\text{m}$,
$F = 20N$, $\omega = 50 \text{rad s}^{-1}$
$\theta = 6.28 \text{rad}$ (for one revolution).
We know that,
Moment of inertia × Angular acceleration
α

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{F \times R}{\frac{MR^2}{2}} = \frac{2F}{MR} \quad [\because \tau = F \times R \text{ and } I_{disc} = \frac{1}{2}MR^2]$$

$$\Rightarrow \alpha = \frac{2 \times 20}{20 \times (0.2)} = 10 \text{rad} / s^2$$

In angular terms, third equation of motion,

$$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$$

$$\Rightarrow (50)^2 = (0)^2 + 2 \times 10 \times \Delta\theta$$

$$\Rightarrow \Delta\theta = \frac{2500}{20}$$

$$\Rightarrow \Delta\theta = 125 \text{rad}$$

$$\therefore \text{Number of revolution} = \frac{\Delta\theta}{\theta} = \frac{125}{6.28} \approx 20$$

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Question69

A force $F = 4\hat{i} + 3\hat{j} + 4\hat{k}$ is applied on an intersection point of $x = 2$ plane and X -axis. The magnitude of torque of this force about a point (2, 3, 4) is ....
(Round off to the nearest integer)
[16 Mar 2021 Shift 2]

Solution:

Given,

Force, $F = 4\hat{i} + 3\hat{j} + 4\hat{k}$
We know that
Torque, $\tau = r \times F$
where, r is the perpendicular distance.

$r = \left(2\hat{i}\right) - \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = -3\hat{j} - 4\hat{k}$

$$\Rightarrow \tau = r \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$\Rightarrow \tau = \hat{i}(-12 + 12) - \hat{j}(0 + 16) + \hat{k}(0 + 12)$

$\Rightarrow \tau = -16\hat{j} + 12\hat{k}$

∴ Magnitude of torque

$|\tau| = \sqrt{(16)^2 + (12)^2}$

$= \sqrt{256 + 144} = \sqrt{400}$

$\Rightarrow |\tau| = 20$

Question70

Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is
[18 Mar 2021 Shift 2]

Options:

A. $\frac{1}{4}\frac{ML^2}{\pi^2}$

B. $\frac{2}{5}\frac{ML^2}{\pi^2}$

C. $\frac{ML^2}{\pi^2}$

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D. $\frac{1}{2} \frac{ML^2}{\pi^2}$

Answer: C

Solution:

Solution:
The uniform wire whose mass is M and length is L and bent into a semicircle of radius r.
The length of the semicircle wire, $L = \pi r$

$\Rightarrow r = \frac{L}{\pi}$

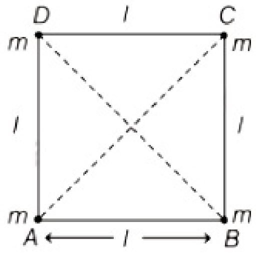
The moment of inertia,

$I = M r^2$

$\Rightarrow I = M\left(\frac{L}{\pi}\right)^2 \Rightarrow I = \frac{ML^2}{\pi^2}$

Question71

Four equal masses, m each are placed at the corners of a square of length (I) as shown in the figure



[16 Mar 2021 Shift 1]

Options:

A. The moment of inertia of the system about an axis passing through A and parallel to DB would be

B. $1ml\,l^2$

C. $2ml^2$

D. $3ml^2$

E. $\sqrt{3}ml^2$

Answer: C

Solution:

Solution:
The given situation can be represented as follows

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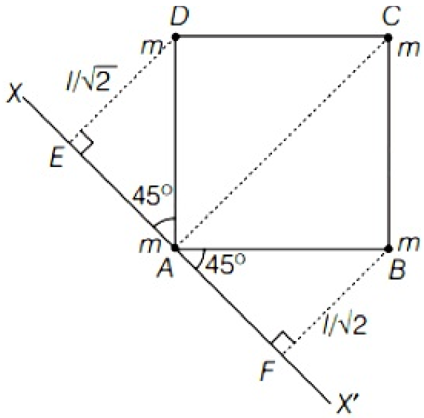

In the above figure, XX′ be the axis which passes through A and is parallel to DB.
∴Moment of inertia of the system
= Mass × (Perpendicular distance from axis $)^2$

$$I = m(AC)^2 + m(ED)^2 + m(FB)^2$$

$$= m(0)^2 + m(I\sqrt{2})^2 + m\left(\frac{1}{\sqrt{2}}\right)^2 + m\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3ml^2$$

Question72

A solid disc of radius 20cm and mass 10kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10s is _____ $\pi \times 10^{-1}$N m.
[25 Jul 2021 Shift 2]

Answer: 4

Solution:

Solution:

$$\tau = \frac{\Delta L}{\Delta t} = \frac{I(\omega_f - \omega_i)}{\Delta t}$$

$$\tau = \frac{\frac{mR^2}{2} \times [0 - \omega]}{\Delta t}$$

$$= \frac{10 \times (20 \times 10^{-2})^2}{2} \times \frac{600 \times \pi}{30 \times 10}$$

$$= 0.4\pi = 4\pi \times 10^{-2}$$

Question73

A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed.
The speed of the two body centre of mass is $\frac{x}{10}$m / s . Then the value of x is_______.

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[25 Jul 2021 Shift 1]

Solution:

Solution:

$p_i = p_f$

$2 \times 4 = 2 \times 1 + m_2 \times v_2$

$m_2 v_2 = 6$ ....(i)

by coefficient of restitution

$1 = \frac{v_2 - 1}{4} \Rightarrow v_2 = 5m / s$

by (i)

$m_2 \times 5 = 6$

$m_2 = 1.2kg$

$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$v_{cm} = \frac{2 \times 1 + 1.2 \times 5}{2 + 1.2} = \frac{8}{3.2} = \frac{25}{10}$

$x = 25$

Question74

A particle of mass 'm' is moving in time 't' on a trajectory given by
$\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$
Where α and β are dimensional constants.
The angular momentum of the particle becomes the same as it was for t = 0 at time t = ______seconds.
[25 Jul 2021 Shift 1]

Answer: 10

$\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$

$\vec{v} = 20\alpha \hat{i} + 5\beta \hat{j}$

$\vec{L} = m(\vec{r} \times \vec{v})$

$= m[10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}] \times [20\alpha t \hat{i} + 5\beta \hat{j}]$

$\vec{L} = m[50\alpha\beta t^2 \hat{k} - 100\alpha\beta(t^2 - 5t)\hat{k}]$

At $t = 0$, $\vec{L} = \vec{0}$

$50\alpha\beta t^2 - 100\alpha\beta(t^2 - 5t) = 0$

$t - 2(t - 5) = 0$

$t = 10sec$

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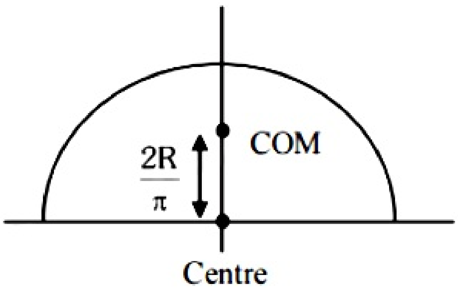
Question75

The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in $x - y$ plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$.
Then, the value of $|x|$ is __________.
[22 Jul 2021 Shift 2]

Answer: 2

Solution:

Solution:
COM of semi-circular ring is at $\frac{2R}{\pi}$



Distance from centre ⇒ x = 2

Question76

A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of ' x ' will be ______.
[20 Jul 2021 Shift 1]

Solution:

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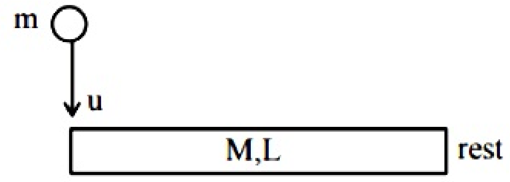
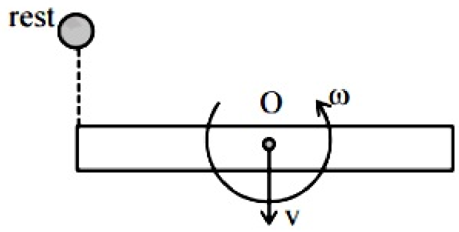


Just before collision



Just after collision

From momentum conservation,

$P_i^0 = P_f$

$mu = Mv$....... (i)

From angular momentum conservation about O,

$mu \cdot \frac{L}{2} = \frac{ML^2}{12}\omega$

$\Rightarrow \omega = \frac{6mu}{ML}$ ....(ii)

From $e = \frac{R.V.S}{R.V.A}$

$1 = \frac{V + \frac{\omega L}{2}}{u}$

$v + \frac{\omega L}{2} = u$

$v + \frac{3mu}{M} = u$

$\frac{mu}{M} + \frac{3mu}{M} = u$

$\frac{4mu}{M} = u$

$\frac{m}{M} = \frac{1}{4}$

$X = 4$

Question77

	List-I		List-II
(a)	Moment of Inertia(MI) of the rod (length L, Mass M, about an axis ⊥ to the rod passing through the midpoint)	(i)	$8ML^2/3$
(b)	Moment of Inertia(MI) of the rod (length L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(ii)	$ML^2/3$
(c)	Moment of Inertia(MI) of the rod (length 2L, Mass M, about an axis ⊥ to the rod passing through its midpoint)	(iii)	$ML^2/12$
(d)	Moment of Inertia(MI) of the rod (Length 2L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(iv)	$2ML^2/3$

Choose the correct answer from the options given below :
[27 Jul 2021 Shift 1]

Options:

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A. (a)–(ii), (b)–(iii), (c)– (i), (d)–(iv)

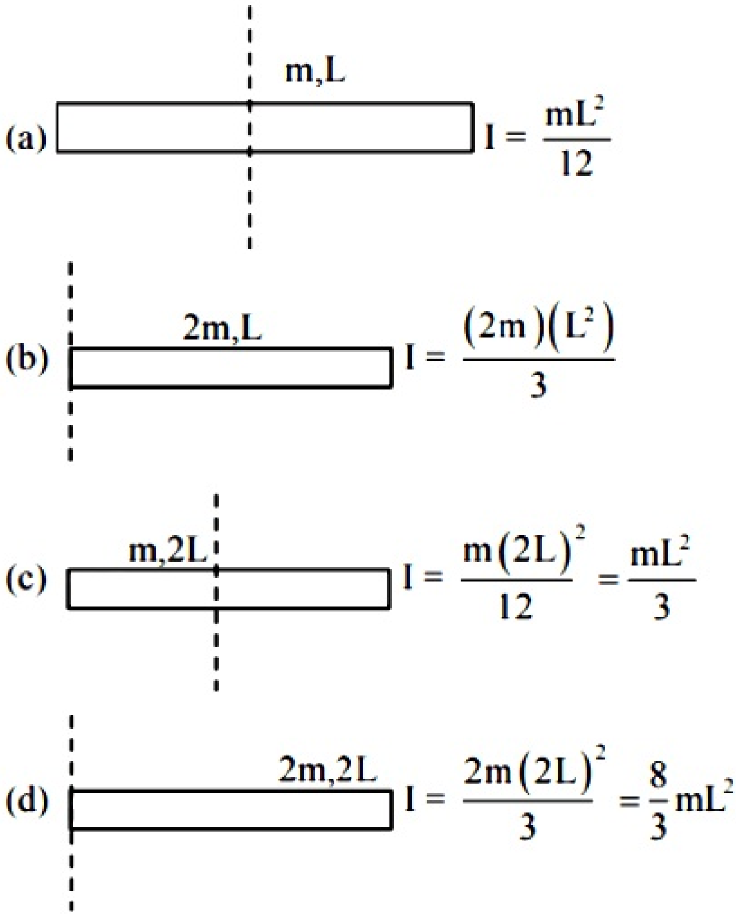
B. (a)–(ii), (b)–(i), (c)– (iii), (d)–(iv)

C. (a)–(iii), (b)–(iv), (c)– (ii), (d)–(i)

D. (a)–(iii), (b)–(iv), (c)– (i), (d)–(ii)

Answer: C

Solution:



Question78

The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is ⊥ to the plane of the disc and passing

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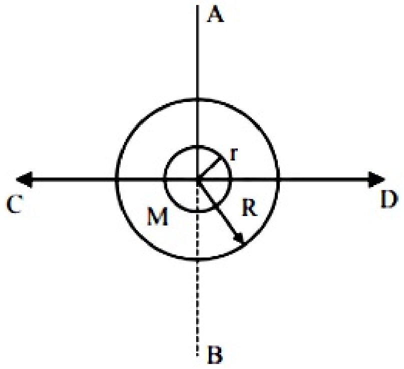
through its centre) of MI of smaller disc around one of its diameters lying on its plane?
Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)



[27 Jul 2021 Shift 1]

Options:

A. $R^2 : r^2$

B. $2r^4 : R^4$

C. $2R^2 : r^2$

D. $2R^4 : r^4$

Answer: D

Solution:

Ratio of moment of inertia $= \dfrac{\frac{1}{2}M R^2}{\frac{1}{4}mr^2}$

$$= \frac{2\sigma\pi R^2 R^2}{\sigma\pi r^2 r^2} = \frac{2R^4}{r^4}$$

Question79

Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A: Moment of inertia of a circular disc of mass ' M ' and radius 'R' about X , Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y and I_z respectively. The respective radii of gyration about all the three axes will be the same.
Reason R : A rigid body making rotational motion has fixed mass and shape. In the light of the above statements, choose the most appropriate answer from the options given below:
[25 Jul 2021 Shift 1]

Options:

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A. Both A and R are correct but R is NOT the correct explanation of A.

B. A is not correct but R is correct.

C. A is correct but R is not correct.

D. Both A and R are correct and R is the correct explanation of A.

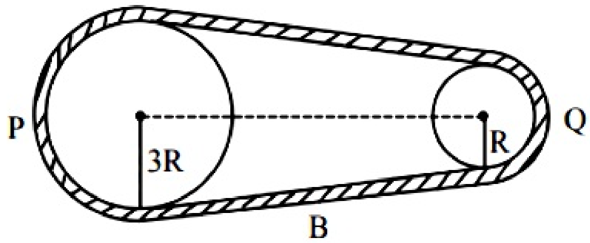
Answer: B

Solution:

Solution:
$I_z = I_x + I_y$ (using perpendicular axis theorem)
& $I = mk^2$ (K : radius of gyration)
so $mK_z^2 = mK_x^2 + mK_y^2$
$K_z^2 = K_x^2 + K_y^2$
so radius of gyration about axes x, y & z won't be same hense asseration A is not correct reason R is correct statement (property of a rigid body)

Question80

In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be ______.



[27 Jul 2021 Shift 2]

Solution:

$$\frac{1}{2}I_1(\omega_1)^2 = \frac{1}{2}I_2(\omega_2)^2$$

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$I_1\left(\frac{v}{3R}\right)^2 = I_2\left(\frac{v}{R}\right)^2$

$\frac{I_1}{I_2} = \frac{9}{1}$

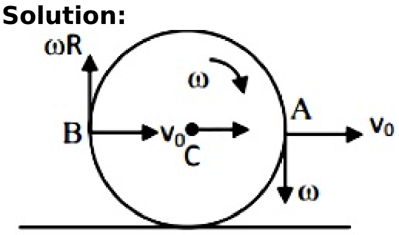
Question81

The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}v_0$. Then the value of x is ______.
[22 Jul 2021 Shift 2]

Answer: 2

Solution:

Solution:



For no slipping $v_0 = \omega R$

Now $v_A = v_B = \sqrt{v_0^2 + (\omega R)^2}$

$= \sqrt{2}v_0$

$\Rightarrow x = 2$

Question82

Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.
The correct statement for this situation is:-
[22 Jul 2021 Shift 2]

Options:

A. The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

B. The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

C. All of them will have same velocity.

D. The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

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Answer: A

Solution:

Solution:

$a = g\sin\theta\, 1 + \frac{I}{mR^2}$

$I_{ring} > I_{solid\ cylinder} > I_{solid\ sphere}$

$\Rightarrow a_{ring} < a_{solid\ cylinder} < a_{solid\ sphere}$

$\Rightarrow v_{ring} < v_{solid\ cylinder} < v_{solid\ sphere}$

Question83

A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is : [20 Jul 2021 Shift 2]

Options:

A. Solid sphere

B. Solid cylinder

C. Hollow cylinder

D. Ring

Answer: B

Solution:

Solution:

$\frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2}mv^2$

$I = \frac{1}{2}mR^2$

Body is solid cylinder

Question84

A circular disc reaches from top to bottom of an inclined plane of length ' L′. When it slips down the plane, it takes time ' t_1′. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be ______.

[20 Jul 2021 Shift 1]

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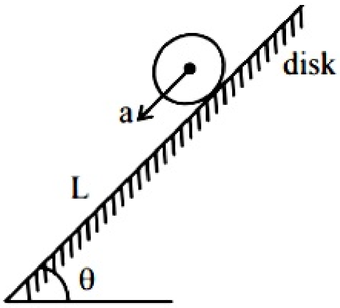
Solution:



If disk slips on inclined plane, then it's acceleration

$a_1 = g\sin\theta$

$L = \frac{1}{2}a_1 t_1^2$

$\Rightarrow t_1 = \sqrt{\frac{2L}{a_1}}$ ....(i)

If disk rolls on inclined plane, its acceleration,

$a_2 = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$

$a_2 = \frac{g\sin\theta}{1 + \frac{mR^2}{2mR^2}}$

$a_2 = \frac{2}{3}g\sin\theta$

Now $L = \frac{1}{2}a_2 \cdot t_2^2$

$\Rightarrow t_2 = \sqrt{\frac{2L}{a_2}}$ ......(ii)

Now $\frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{3}{2}}$

$\Rightarrow x = 2$

Question85

Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity of the centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is _____.

[20 Jul 2021 Shift 2]

Answer: 3

Solution:

Solution:

I in both cases is about point of contact

Ring

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$mgh = \frac{1}{2} I \omega^2$

$mgh = \frac{1}{2}(2mR^2)\frac{v_R^2}{R^2}$

$v_R = \sqrt{gh}$

Solid cylinder

$mgh = \frac{1}{2} I \omega^2$

$mgh = \frac{1}{2}\left(\frac{3}{2}mR^2\right)\frac{v_C^2}{R^2}$

$v_C = \sqrt{\frac{4gh}{3}}$

$\frac{v_R}{v_C} = \frac{\sqrt{3}}{2}$

Question86

A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is___.
[20 Jul 2021 Shift 2]

Options:

Answer: 32

Solution:

Solution:

Given, initial angular speed,

$\omega_0 = 600rpm = 10rps$

Final angular speed,

$\omega_F = 1800rpm = 30rps$

Time, $t = 10s$

∵ We know that

$\omega_F = \omega_0 + \alpha t$

$\Rightarrow 30 = 10 + \alpha \times 10$

$\Rightarrow \frac{30 - 10}{10} = \alpha$

$\Rightarrow \alpha = \frac{20}{10} = 2rps^2$

Angular displacement is calculated by the rotational equation of motion as

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

where, θ is the angular displacement.

$\Rightarrow \theta = 10 \times 10 + \frac{1}{2} \times 2 \times (10)^2$

$\Rightarrow \theta = 100 + 100$

$\Rightarrow \theta = 200rad$

Number of rotations

$= \frac{\theta}{2\pi} = \frac{200}{2\pi} = 31.8 \approx 32$

Question87

Angular momentum of a single particle moving with constant speed along circular path

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[31 Aug 2021 Shift 1]

Options:

A. changes in magnitude but remains same in the direction

B. remains same in magnitude and direction

C. remains same in magnitude but changes in the direction

D. is zero

Answer: B

Solution:

Solution:
As we know that,
Angular momentum, $L = r \times p = mvr \sin\theta$
where, m = mass,v = velocity,r = radius,
$\theta =$ angle between v and r = 90° (for circular motion) Since, m, v, r
∴ L will always remain same in magnitude and direction.

Question88

A system consists of two identical spheres each of mass 1.5 kg and radius 50 cm at the end of light rod. The distance between the centres of the two spheres is 5m. What will be the moment of inertia of the system about an axis perpendicular to the rod passing through its mid-point? [31 Aug 2021 Shift 2]

Options:

A. 18.75kgm^2

B. $1.905 \times 10^5\text{kgm}^2$

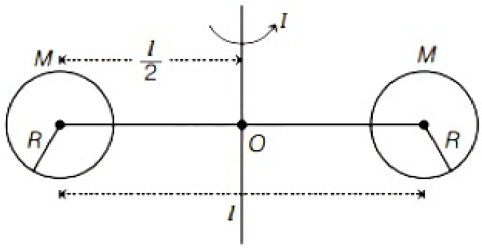
C. 19.05kgm^2

D. $1.875 \times 10^5\text{kgm}^2$

Answer: C

Solution:

Solution:
Given, mass of each sphere, M= 1.5 kg
Radius of each sphere, $R = 50\text{ cm} = 50 \times 10^{-2}\text{m}$
Distance between centre of spheres, l = 5 m



By using parallel axis theorem, moment of inertia of system is

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$I = 2\left[\frac{2}{5}MR^2 + \frac{Ml^2}{4}\right]$

$I = 2M\left[\frac{2}{5}R^2 + \frac{l^2}{4}\right]$

Substituting the values, we get

$I = 2 \times 1.5\left[\frac{2}{5} \times (50 \times 10^{-2})^2 + \frac{(5)^2}{4}\right]$

$= 19.05\ \mathrm{kg - m^2}$

Question89

Moment of inertia of a square plate of side / about the axis passing through one of the corner and perpendicular to the plane of square plate is given by [27 Aug 2021 Shift 1]

Options:

A. $\frac{MI^2}{6}$

B. MI^2

C. $\frac{MI^2}{12}$

D. $\frac{2}{3}MI^2$

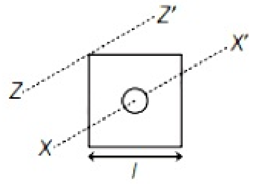
Answer: D

Solution:

Solution:

Let the length of side of square plate is l.

The diagram of square plate having an axis passing from centre as $X - X'$ and an axis passing through one of the corner and perpendicular to plane of square plate $Z - Z'$ is shown below.



The distance between the centre of square plate and the corner of square plate is

$d = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$

We know that, moment of inertia of square plate about centre,

$I_{XX'} = \frac{MI^2}{6}$

By parallel axis theorem, moment of inertia of square plate about axis $Z - Z'$ will be

$I_{ZZ'} = I_{XX'} + M\left(\frac{1}{\sqrt{2}}\right)^2$

$I_{ZZ'} = \frac{MI^2}{6} + \frac{MI^2}{2}$

$= MI^2\left[\frac{1}{6} + \frac{1}{2}\right]$

$I_{ZZ'} = \frac{2MI^2}{3}$

Thus, the moment of inertia about an axis passing through corner of square plate and perpendicular to plane of plate is $2\frac{MI^2}{3}$.

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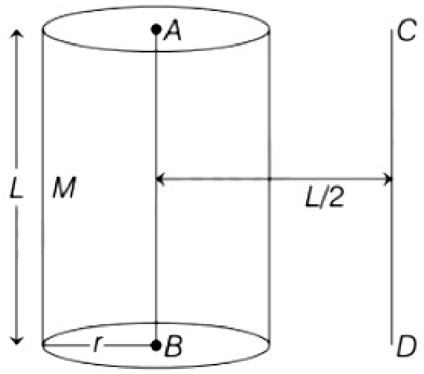
Question90

The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used, if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is $2.7\ kg\ m^2$.



[26 Aug 2021 Shift 2]

Options:

A. $14.9\ kg / m^3$

B. $7.5 \times 10^9\ kg / m^3$

C. $7.5 \times 10^2\ kg / m^3$

D. $1.49 \times 10^2\ kg / m^3$

Answer: D

Solution:

Solution:
Given, the length of cylinder,
L = 80 cm = 0.8 m
The radius of cylinder, r = 20 cm = 0.2 m
The moment of inertia about CD, $I_{CD} = 2.7\ kg - m^2$
Now, the moment of inertia of cylinder about its axis AB is
$I_{AB} = \frac{1}{2}Mr^2$
By parallel axis theorem, the moment of inertia of cylinder about CD will be calculated as
$I_{CD} = I_{AB} + M\left(\frac{L}{2}\right)^2$
$\Rightarrow 2.7 = \frac{Mr^2}{2} + ML^2 4$
$\Rightarrow 2.7 = M\left[\frac{(0.2)^2}{2} + (0.8)^2 4\right]$
$\Rightarrow M = 15\ kg$
Density of cylinder is given as
$\rho = \frac{M}{V} = \frac{M}{\pi r^2 L}$
$= \frac{15}{\pi(0.2)^2(0.8)} = 149.2 kgm^{-3}$
$= 1.49 \times 10^2\ kg\ m^{-3}$
Thus, the density of cylinder is $1.49 \times 10^2 kgm^{-3}$.

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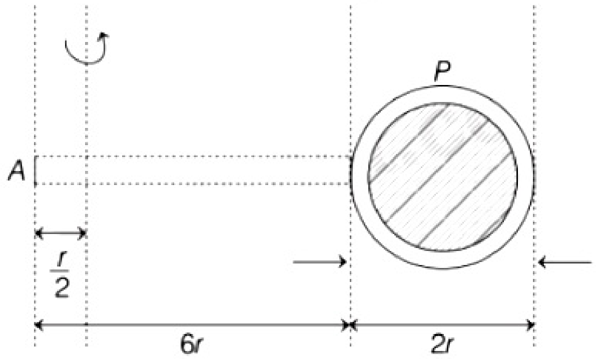
Question91

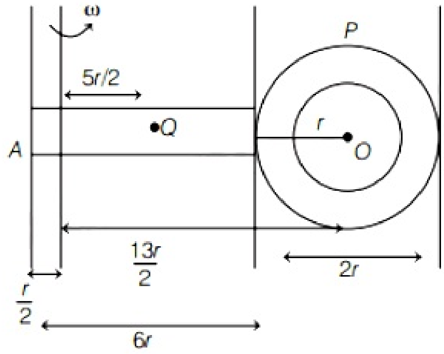
Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\frac{r}{2}$ distance from the end A of the handle will be .... Mr^2.
[26 Aug 2021 Shift 1]

Solution:

Solution:
The given situation is shown in the following figure



The moment of inertia of the racket about an axis perpendicular to the handle and in the plane of ring at, $\frac{r}{2}$ distance from end A of the handle is given by

$I = I_{\text{due to handle}} + I_{\text{due to disc}}$ ...(i)

$I_{\text{due to handle}} = \frac{1}{12}M(6r)^2 + M\left(\frac{5r}{2}\right)^2$

$= \frac{1}{12}M36r^2 + \frac{25Mr^2}{4}$

$= 3Mr^2 + \frac{25}{4}Mr^2$

$= Mr^2\left(\frac{37}{4}\right)$

$= \frac{37}{4}Mr^2$

$I_{\text{due to disc}} = Mr^2 2 + M\left(\frac{13r}{2}\right)^2$

$= \frac{Mr^2}{2} + \frac{169Mr^2}{4}$

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$= \frac{171Mr^2}{4}$

∴ From Eq. (i), we get

$I = I_{\text{dueto handle}} + I_{\text{due to disc}}$

$= \frac{37}{4}Mr^2 + \frac{171Mr^2}{4}$

$= \frac{208}{4}Mr^2$

$= 52Mr^2$

Question92

Two discs have moments of inertia I_1 and I_2 about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, ω_1 and ω_2 respectively and are brought into contact face to face with their axes of rotation co-axial. The loss in kinetic energy of the system in the process is given by
[27 Aug 2021 Shift 2]

Options:

A. $\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$

B. $\frac{(I_1 - I_2)^2 \omega_1 \omega_2}{2(I_1 + I_2)}$

C. $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$

D. $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

Answer: C

Solution:

Solution:
Given, moment of inertia of two discs are I_1, I_2 .

Their initial angular velocities is ω_1 and ω_2 .

Let final angular velocity is ω . By using law of conservation of angular momentum,

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$\omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$

∴ Change in kinetic energy $\triangle KE$

$= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2 - \frac{1}{2}(I_1 + I_2)\omega^2$

Question93

A 2 kg steel rod of length 0.6m is clamped on a table vertically at its

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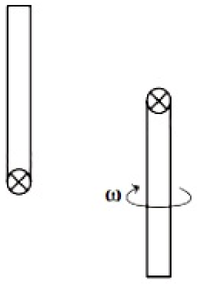
lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity. Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is .... ms^{-1}.
(Take, $g = 10ms^{-2}$)
[1 Sep 2021 Shift 2]

Answer: 6

Solution:

Solution:
Given, the mass of the steel rod, m = 2 kg
The length of the steel rod, l = 0.6 m
According to question, the diagram can be as shown below



By using the energy conservation,
$$mgl = \frac{1}{2}I\omega^2$$
$$mgl = \frac{1}{2} \times \frac{ml^2}{3} \times \omega^2$$
$$\Rightarrow \omega = \frac{\sqrt{6g}}{l}$$
As we know the relation between the linear speed and angularspeed,
$v = \omega r = \omega l = \sqrt{6gl}$
Substituting the values in the above equation, we get
$v = \sqrt{6 \times 10 \times 0.6} = 6 \text{ m / s}$
Hence, the speed of the free end of the rod when it passes throughits lowest position is 6 m/s.

Question94

A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:
[9 Jan. 2020 II]

Options:

A. $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

B. $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

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C. $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$

D. $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$

Answer: B

Solution:

Solution:

Given,

Linear mass density, $\rho(x) = a + b\left(\frac{x}{L}\right)^2$

$X_{CM} = \frac{\int x\,dm}{\int dm}$

$\int dm = \int_0^L \rho(x)dx$

$= \int_0^L \left[a + b\left(\frac{x}{L}\right)^2\right]dx = aL + \frac{bL}{3}$

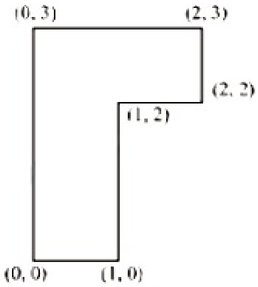
$\int_0^L x\,dm = \int_0^L \left(ax + \frac{bx^3}{L^2}\right)dx = \left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)$

$\therefore X_{CM} = \frac{\left(\frac{aL^2}{2} + \frac{bL^2}{4}\right)}{aL + \frac{bL}{3}}$

$\Rightarrow X_{CM} = \frac{3L}{4}\left(\frac{2a+b}{3a+b}\right)$

Question95

The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are:



[8 Jan. 2020 I]

Options:

A. (1.25m, 1.50m)

B. (0.75m, 1.75m)

C. (0.75m, 0.75m)

D. (1m, 1.75m)

Answer: B

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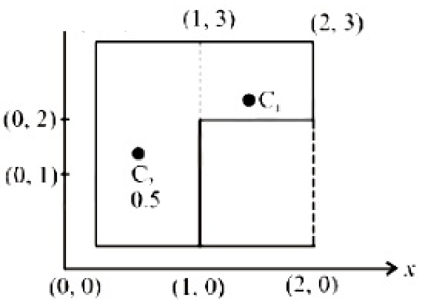
Solution:



For given Lamina

xy$m_1 = 1$, $C_1 = (1.5, 2.5)$

$m_2 = 3$, $C_2 = (0.5, 1.5)$

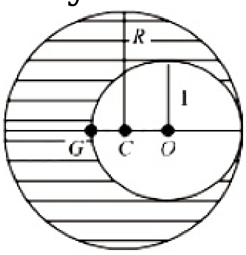
$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1.5 + 1.5}{4} = 0.75$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{2.5 + 4.5}{4} = 1.75$$

∴ Coordinate of centre of mass of flag shaped lamina (0.75,1.75)

Question96

As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation:



[8 Jan. 2020 II]

Options:

A. $(R^2 + R + 1)(2 - R) = 1$

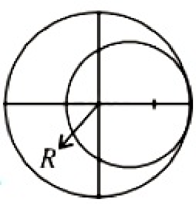
B. $(R^2 - R - 1)(2 - R) = 1$

C. $(R^2 - R + 1)(2 - R) = 1$

D. $(R^2 + R - 1)(2 - R) = 1$

Answer: A

Solution:



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Mass of sphere = volume of sphere x density of sphere $= \frac{4}{3}\pi R^3\rho$

Mass of cavity $M_{\text{cavity}} = \frac{4}{3}\pi(1)^3\rho$

Mass of remaining

$M_{(\text{Remaining})} = \frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi(1)^3\rho$

Centre of mass of remaining part,

$$X_{COM} = \frac{M_1 r_1 + M_2 r_2}{M_1 + M_2}$$

$$\Rightarrow -(2 - R) = \frac{\left[\frac{4}{3}\pi R^3\rho\right]0 + \left[\frac{4}{3}\pi(1)^3(-\rho)\right][R - 1]}{\frac{4}{3}\pi R^3\rho + \frac{4}{3}\pi(1)^3(-\rho)}$$

$$\Rightarrow (R - 1)(R^3 - 1) = 2 - R$$

$$\Rightarrow \frac{(R - 1)}{(R - 1)(R^2 + R + 1)} = 2 - R$$

$$\Rightarrow (R^2 + R + 1)(2 - R) = 1$$

Question97

Three point particles of masses 1.0kg, 1.5kg and 2.5kg are placed at three corners of a right angle triangle of sides 4.0cm, 3.0cm and 5.0cm as shown in the figure. The center of mass of the system is at a point: [7 Jan. 2020 I]

Options:

A. 0.6cm right and 2.0cm above 1kg mass

B. 1.5cm right and 1.2cm above 1kg mass

C. 2.0cm right and 0.9cm above 1kg mass

D. 0.9cm right and 2.0cm above 1kg mass

Answer: D

Solution:

Solution:

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$X_{cm} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{1 + 1.5 + 2.5} = \frac{1.5 \times 3}{5} = 0.9\text{cm}$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$Y_{cm} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{1 + 1.5 + 2.5} = \frac{2.5 \times 4}{5} = 2\text{cm}$$

Hence, centre of mass of system is at point (0.9,2)

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Question98

A spring mass system (mass m, spring constant k and natural length l) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, ($k >> m\omega^2$) the relative change in the length of the spring is best given by the option:
[9 Jan. 2020 II]

Options:

A. $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$

B. $\frac{2m\omega^2}{k}$

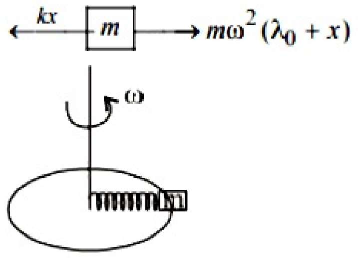
C. $\frac{m\omega^2}{k}$

D. $\frac{m\omega^2}{3k}$

Answer: C

Solution:

Solution:
Free body diagram in the frame of disc



$\therefore m\omega^2(l_0 + x) = kx$

$\Rightarrow x = \frac{ml_0\omega^2}{k - m\omega^2}$

For $k >> m\omega^2$

$\Rightarrow \frac{x}{l_0} = \frac{m\omega^2}{k}$

Question99

A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length l . The other end is fixed. The system is given an angular speed omega about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is:
[8 Jan. 2020 I]

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Options:

A. $\frac{ml\omega^2}{k-\omega m}$

B. $\frac{ml\omega^2}{k-m\omega^2}$

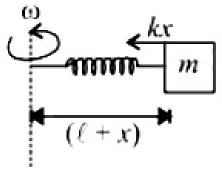
C. $\frac{ml\omega^2}{k+m\omega^2}$

D. $\frac{ml\omega^2}{k+m\omega}$

Answer: B

Solution:

Solution:



At elongated position (x)

$F_{radial} = \frac{mv^2}{r} = mr\omega^2$

$\therefore kx = m(l + x)\omega^2$

$(\because r = l + x \text{ here })$

$kx = ml\omega^2 + mx\omega^2$

$\therefore x = \frac{ml\omega^2}{k-m\omega^2}$

Question100

Consider a uniform rod of mass $M = 4m$ and length l pivoted about its centre. A mass m moving with velocity v makingangle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is:
[8 Jan. 2020 I]

Options:

A. $\frac{3}{7\sqrt{2}}\frac{v}{l}$

B. $\frac{3v}{7l}$

C. $\frac{3\sqrt{2}}{7}\frac{v}{l}$

D. $\frac{4v}{7l}$

Answer: C

Solution:

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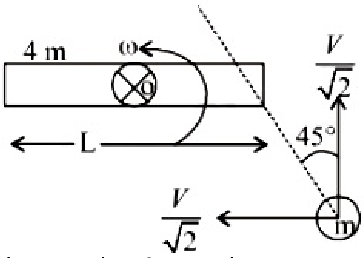


About point O angular momentum

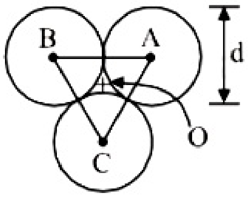
$L_{initial} = L_{final}$

$\Rightarrow \frac{mV}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4}\right] \times \omega$

$\therefore \omega = \frac{6V}{7\sqrt{2}L} = \frac{3\sqrt{2}V}{7L}$

Question101

Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio $\frac{I_0}{I_A}$ of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is:



[9 Jan. 2020 I]

Options:

A. $\frac{13}{23}$

B. $\frac{15}{13}$

C. $\frac{23}{13}$

D. $\frac{13}{15}$

Answer: A

Solution:

Solution:
Moment of inertia,

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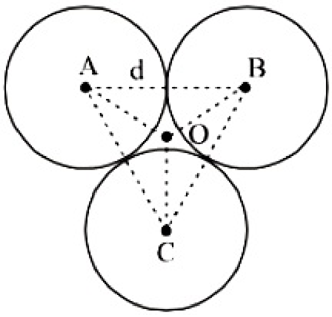


$I_1 = \frac{2}{5}m\left(\frac{d}{2}\right)^2 + m(AO)^2$

and $AO = \frac{d}{\sqrt{3}}$

Moment of inertia about ' O '

$I_0 = 3I_1 = 3\left[\frac{2}{5}m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{\sqrt{3}}\right)^2\right]$

$\Rightarrow I_0 = \frac{13}{10}Md^2$

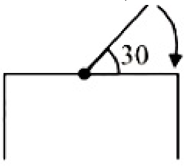
And $I_A = 2\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + Md^2\right] + \frac{2}{5}M\left(\frac{d}{2}\right)^2$

$\Rightarrow I_A = \frac{23}{10}Md^2$

$\therefore \frac{I_O}{I_A} = \frac{\frac{13}{10}Md^2}{\frac{23}{10}Md^2} = \frac{13}{23}$

Question102

One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30° from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}s^{-1}$, where n is an integer. The value of n is ________.



[9 Jan. 2020 I]

Solution:

Here, length of bar, $l = 1m$

angle, $\theta = 30°$

$\Delta PE = \Delta KE$ or $mgh = \frac{1}{2}I\omega^2$

$\Rightarrow (mg)\frac{l}{2}\sin 30° = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$

$\Rightarrow mg\frac{l}{2} \times \frac{1}{2} = \frac{1}{2}\left(\frac{ml^2}{3}\right)\omega^2$

$\Rightarrow \omega = \sqrt{15} rad / s$

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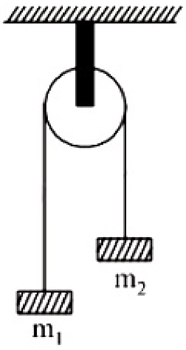
Question103

A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and $m_2 (m_1 > m_2)$ are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is:



[9 Jan. 2020 II]

Options:

A. $\left[\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + 1}\right]^{1/2}$

B. $\left[\frac{2(m_1 + m_2)gh}{(m_1 + m_2)R^2 + 1}\right]^{1/2}$

C. $\left[\frac{(m_1 - m_2)}{(m_1 + m_2)R^2 + 1}\right]^{1/2} gh$

D. $\left[\frac{(m_1 + m_2)}{(m_1 + m_2)R^2 + 1}\right]^{1/2} gh$

Answer: A

Solution:

Solution:
Using principal of conservation of energy

$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\omega^2$

$\Rightarrow (m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)(\omega R)^2 + \frac{1}{2}I\omega^2 \ (\because v = \omega R)$

$\Rightarrow (m_1 - m_2)gh = \frac{\omega^2}{2}[(m_1 + m_2)R^2 + I]$

$\Rightarrow \omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R^2 + I}}$

Question104

As shown in the figure, a bob of mass m is tied by a massless string whose other end portion is wound on a fly wheel (disc) of radius r and

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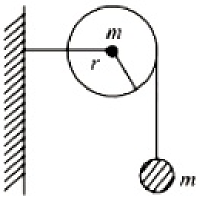
mass m. When released from rest the bob starts falling vertically. When it has covered a distance of h, the angular speed of the wheel will be:



[7 Jan. 2020 I]

Options:

A. $\frac{1}{r}\sqrt{\frac{4gh}{3}}$

B. $r\sqrt{\frac{3}{2gh}}$

C. $\frac{1}{r}\sqrt{\frac{2gh}{3}}$

D. $r\sqrt{\frac{3}{4gh}}$

Answer: A

Solution:

Solution:
When the bob covered a distance ' h '
Using $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\,\omega^2$
$= \frac{1}{2}m(\omega r)^2 + \frac{1}{2} \times \frac{mr^2}{2} \times \omega^2$ ($\because v = \omega r$ no slipping)
$\Rightarrow mgh = \frac{3}{4}m\omega^2 r^2$
$\Rightarrow \omega = \sqrt{\frac{4gh}{3r^2}} = \frac{1}{r}\sqrt{\frac{4gh}{3}}$

Question105

The radius of gyration of a uniform rod of length l , about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is:
[7 Jan. 2020 I]

Options:

A. $\frac{1}{4}l$

B. $\frac{1}{8}l$

C. $\sqrt{\frac{7}{48}}l$

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D. $\sqrt{\frac{3}{8}}l$

Answer: C

Solution:

Solution:

Moment inertia of the rod passing through a point $\frac{l}{4}$ away from the centre of the rod

$I = Ig + ml^2$

$\Rightarrow I = \frac{MI^2}{12} + M \times \left(\frac{I^2}{16}\right) = \frac{7MI^2}{48}$

Using $I = MK^2 = \frac{7MI^2}{48}$ ($K =$ radius of gyration)

$\Rightarrow K = \sqrt{\frac{7}{48}}I$

Question106

Mass per unit area of a circular disc of radius a depends on the distance r from its centre as σ(r) = A + Br. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:
[7 Jan. 2020 II]

Options:

A. $2\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$

B. $2\pi a^4\left(\frac{aA}{4} + \frac{B}{5}\right)$

C. $\pi a^4\left(\frac{A}{4} + \frac{aB}{5}\right)$

D. $2\pi a^4\left(\frac{A}{4} + \frac{B}{5}\right)$

Answer: A

Solution:

Solution:
Given,
mass per unit area of circular disc, $\sigma = A + Br$
Area of the ring $= 2\pi r dr$
Mass of the ring, $dm = \sigma 2\pi r dr$
Moment of inertia,
$I = \int dm r^2 = \int \sigma 2\pi r dr . r^2$

$\Rightarrow I = 2\pi \int_0^a (A + Br) r^3 dr = 2\pi\left[\frac{Aa^4}{4} + \frac{Ba^5}{5}\right]$

$\Rightarrow I = 2\pi a^4\left[\frac{A}{4} + \frac{Ba}{5}\right]$

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Question107

A uniform sphere of mass 500g rolls without slipping on a plane horizontal surface with its centre moving at a speed of 5.00cm / s. Its kinetic energy is:
[8 Jan. 2020 II]

Options:

A. 8.75×10^{-4}J

B. 8.75×10^{-3}J

C. 6.25×10^{4}J

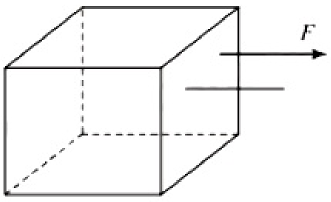
D. 1.13×10^{-3}J

Answer: A

Solution:

Solution:
K . E of the sphere = translational K . E + rotational K . E
$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
Where, I = moment of inertia,
ω = Angular, velocity of rotation
m = mass of the sphere
v = linear velocity of centre of mass of sphere
$\because$ Moment of inertia of sphere $I = \frac{2}{5}mR^2$
$\therefore K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \omega^2$
$\Rightarrow K.E = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \left(\frac{v}{R}\right)^2 \left(\because \omega = \frac{v}{R}\right)$
$\Rightarrow KE = \frac{1}{2}\left(\frac{2}{5}mR^2 + mR^2\right)\left(\frac{v}{R}\right)^2$
$\Rightarrow KE = \frac{1}{2}mR^2 \times \frac{7}{5} \times \frac{v^2}{R^2} = \frac{7}{10} \times \frac{1}{2} \times \frac{25}{10^4}$
$\Rightarrow KE = \frac{35}{4} \times 10^{-4}$ joule
$\Rightarrow KE = 8.75 \times 10^{4}$ joule

Question108



Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is μ = 0.4, the maximum possible value of 100 × $\frac{b}{a}$ for box not to topple before moving is ______.

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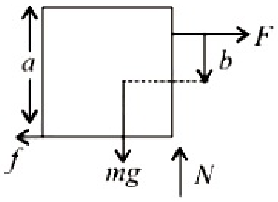
[NA 7 Jan. 2020 II]

Solution:



For the box to be slide

$F = \mu mg = 0.4mg$

For no toppling $F\left(\frac{a}{2}+b\right) \le mg\frac{a}{2}$

$\Rightarrow \& 0.4mg\left(\frac{a}{2}+b\right) \le mg\frac{a}{2}$

$\Rightarrow 0.2a + 0.4b \le 0.5a$

$\Rightarrow \frac{b}{a} \le \frac{3}{4}$

i.e. $b \le 0.75a$ but this is not possible.

As the maximum value of b can be equal to $0.5a$.

$\Rightarrow \frac{100b}{a} = 50$

Question109

The centre of mass of a solid hemisphere of radius 8cm is xcm from the centre of the flat surface. Then value of x is______.
[NA Sep. 06, 2020 (II)]

Answer: 3

Solution:

Centre of mass of solid hemisphere of radius R lies at a distance $\frac{3R}{8}$ above the centre of flat side of hemisphere.

$\therefore h_{cm} = \frac{3R}{8} = \frac{3\times 8}{8} = 3cm$

Question110

A square shaped hole of side $l = \frac{a}{2}$ is carved out at a distance $d = \frac{a}{2}$ from

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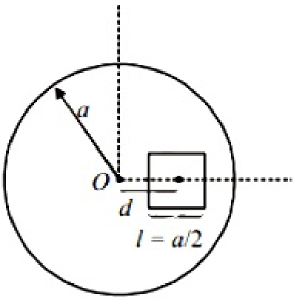
the centre ' O ' of a uniform circular disk of radius a. If the distance of the centre of mass of the remaining portion from O is $-\frac{a}{X}$, value of X (to the nearest integer) is ______.



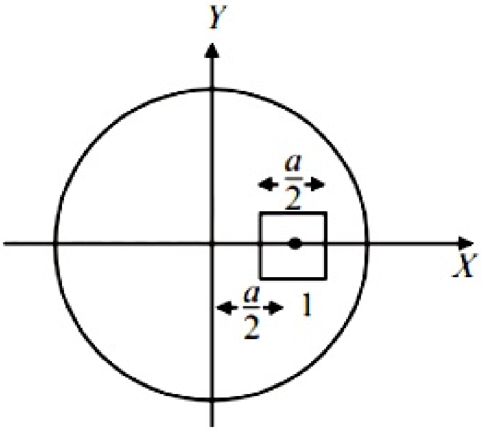
[NA Sep. 02, 2020 (II)]

Solution:

Let σ be the mass density of circular disc.
Original mass of the disc, $m_0 = \pi a^2 \sigma$

nass, $m = \frac{a^2}{4}\sigma$

Remaining, mass, $m' = \left(\pi a^2 - \frac{a^2}{4}\right)\sigma = a^2\left(\frac{4\pi - 1}{4}\right)\sigma$



New position of centre of mass

$$X_{CM} = \frac{m_0 x_0 - mx}{m_0 - m} = \frac{\pi a^2 \times 0 - \frac{a^2}{4} \times \frac{a}{2}}{\pi a^2 - \frac{a^2}{4}}$$

$$= \frac{-a^3 / 8}{\left(\pi - \frac{1}{4}\right)a^2} = \frac{-a}{2(4\pi - 1)} = \frac{-a}{8\pi - 2} = -\frac{a}{23}$$

$\therefore x = 23$

Question111

A bead of mass m stays at point P(a, b) on a wire bent in the shape of a parabola $y = 4Cx^2$ and rotating with angular speed ω (see figure). The value of ω is (neglect friction):

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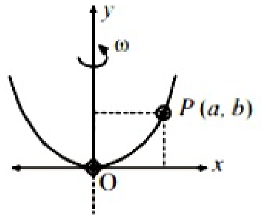


[Sep. 02, 2020 (I)]

Options:

A. $2\sqrt{2gC}$

B. $2\sqrt{gC}$

C. $\sqrt{\frac{2gC}{ab}}$

D. $\sqrt{\frac{2g}{C}}$

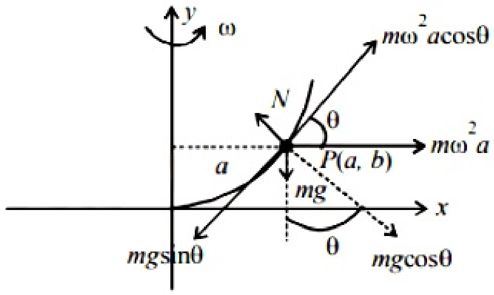
Answer: A

Solution:

Solution:

$y = 4Cx^2 \Rightarrow \frac{dy}{dx} = \tan\theta = 8Cx$

At P, $\tan\theta = 8Ca$



For steady circular motion

$m\omega^2 a\cos\theta = mg\sin\theta$

$\Rightarrow \omega = \sqrt{\frac{g\tan\theta}{a}}$

$\therefore \omega = \sqrt{\frac{g \times 8aC}{a}} = 2\sqrt{2gC}$

Question112

A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5cm and the angular speed of rotation is $\omega\, rad\, s^{-1}$. The difference in the height, h (in cm) of liquid at the centre of vessel and at the side will be :

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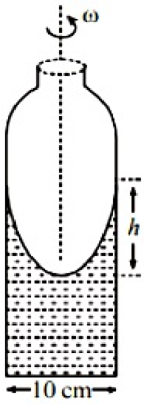


[Sep. 02, 2020 (I)]

Options:

A. $\frac{2\omega^2}{25g}$

B. $\frac{5\omega^2}{2g}$

C. $\frac{25\omega^2}{2g}$

D. $\frac{2\omega^2}{5g}$

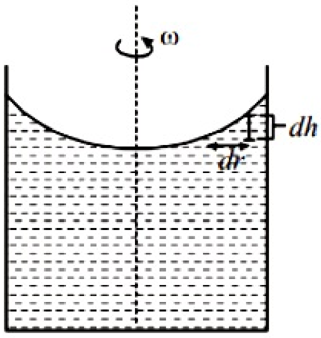
Answer: C

Solution:

Solution:

Here, $\rho d r\omega^2 r = \rho g d h$

$\Rightarrow \omega^2 \int_0^R r d r = g \int_0^R d h$



$\Rightarrow \frac{\omega^2 R^2}{2} = gh$ (Given R = 5cm)

$\therefore h = \frac{\omega^2 R^2}{2g} = \frac{25\omega^2}{2g}$

Question113

Four point masses, each of mass m, are fixed at the corners of a square of side l . The square is rotating with angular frequency omega, about an axis passing through one of the corners of the square and parallel to its diagonal, as shown in the figure. The angular momentum of the square about this axis is :

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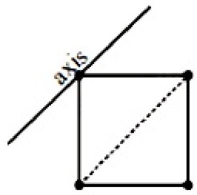


[Sep. 06, 2020 (I)]

Options:

A. $ml^2\omega$

B. $4ml^2\omega$

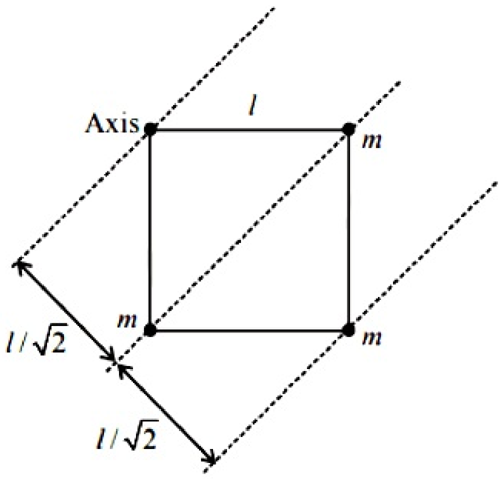
C. $3ml^2\omega$

D. $2ml^2\omega$

Answer: C

Solution:

Solution:
Angular momentum, $L = I\omega$



$I = m(0)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}l)^2$

$= \frac{2ml^2}{2} + 2ml^2 = 3ml^2$

Angular momentum $L = I\omega = 3ml^2\omega$

Question114

A thin rod of mass 0.9kg and length 1m is suspended, at rest, from one end so that it can freely oscillate in the vertical plane. A particle of move 0.1kg moving in a straight line with velocity 80m / s hits the rod at its bottom most point and sticks to it (see figure). The angular speed (in rad/s) of the rod immediately after the collision will be ________.
[NA Sep. 05, 2020 (II)]

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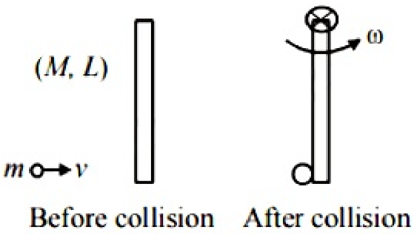
Answer: 20

Solution:

Solution:



Using principal of conservation of angular momentum we have

$\vec{L}_i = \vec{L}_f \Rightarrow mvL = I\omega$

$\Rightarrow mvL = \left(\frac{ML^2}{3} + mL^2\right)\omega$

$\Rightarrow 0.1 \times 80 \times 1 = \left(\frac{0.9 \times 1^2}{3} + 0.1 \times 1^2\right)\omega$

$\Rightarrow 8 = \left(\frac{3}{10} + \frac{1}{10}\right)\omega \Rightarrow 8 = \frac{4}{10}\omega$

$\Rightarrow \omega = 20 \text{ rad / sec}.$

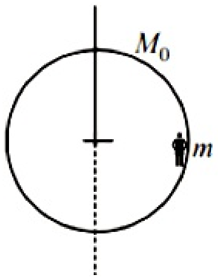
Question115

A person of 80kg mass is standing on the rim of a circular platform of mass 200kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre _______.
[NA Sep. 03, 2020 (I)]

Answer: 9

Solution:

Solution:
Here $M_0 = 200\text{kg}, m = 80\text{kg}$
Using conservation of angular momentum, $L_i = L_f$



$I_1\omega_1 = I_2\omega_2$

$I_1 = (I_M + I_m) = \left(\frac{M_0R^2}{2} + mR^2\right)$

$I_2 = \frac{1}{2}M_0R^2$ and $\omega_1 = 5\text{rpm}$

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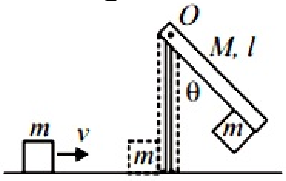
$\therefore \omega_2 = \left(\frac{M_0R^2}{2} + mR^2\right) \times \frac{5}{\frac{M_0R^2}{2}}$

$= \frac{5R^2}{R^2} \times \frac{(80 + 100)}{100} = 9\text{rpm}$

Question116

A block of mass m = 1kg slides with velocity v = 6m / s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2kg, and length l = 1m, the value of θ is approximately: (take $g = 10m/s^2$)



[Sep. 03, 2020 (I)]

Options:

A. 63°

B. 55°

C. 69°

D. 49°

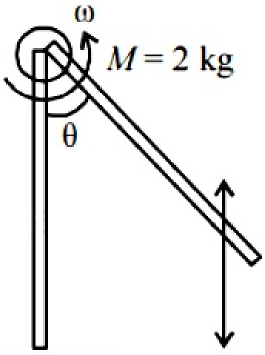
Answer: A

Solution:

Solution:
Using conservation of angular momentum

$mvl = \left(ml^2 + \frac{2ml^2}{3}\right)\omega \Rightarrow mvl = \frac{5}{3}ml^2\omega \Rightarrow \omega = \frac{3v}{5l}$

or, $\omega = \frac{3 \times 6}{5 \times 1} = \frac{18}{5}$ rad / s



$m = 1$ kg

Now using energy conservation, after collision

$\frac{1}{2}I\omega^2 = 2mg\frac{l}{2}(1 - \cos\theta) + mgl(1 - \cos\theta)$

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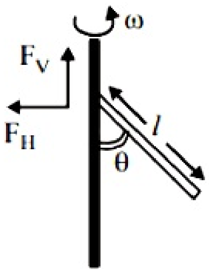
$\Rightarrow \frac{1}{2}\left(\frac{5}{3}ml^2\right)\frac{9v^2}{25l^2} = 2mgl(1-\cos\theta)$

$\Rightarrow \frac{3}{5\times 2}mv^2 = 2mgl(1-\cos\theta)$

$\frac{3}{10}\times\frac{36}{2\times 10} = 1-\cos\theta \Rightarrow 1-\frac{27}{50} = \cos\theta$

or, $\cos\theta = \frac{23}{50} \therefore \theta \simeq 63°$

Question117



A uniform rod of length ' l ' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2\sin\theta\cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :
[Sep. 03, 2020 (II)]

Options:

A. $\cos\theta = \frac{2g}{3l\omega^2}$

B. $\cos\theta = \frac{g}{2l\omega^2}$

C. $\cos\theta = \frac{g}{l\omega^2}$

D. $\cos\theta = \frac{3g}{2l\omega^2}$

Answer: D

Solution:

Solution:
Vertical force $= mg$

Horizontal force = Centripetal force $= m\omega^2\frac{l}{2}\sin\theta$

Torque due to vertical force $= mg\frac{l}{2}\sin\theta$

Torque due to horizontal force $= m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta$

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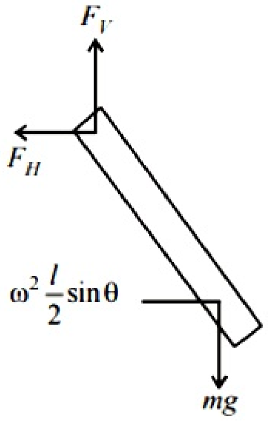
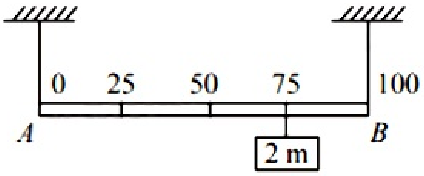


Net Torque = Angular momentum

$mg\frac{l}{2}\sin\theta - m\omega^2\frac{l}{2}\sin\theta\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$

$\Rightarrow \cos\theta = \frac{3}{2}\frac{g}{\omega^2 l}$

Question118



Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2m hung at a distance of 75cm from A. The tension in the string at A is:
[Sep. 02, 2020 (I)]

Options:

A. 0.5mg

B. 2mg

C. 0.75mg

D. 1mg

Answer: D

Solution:

Solution:
Net torque, τ_{net} about B is zero at equilibrium

$\therefore T_A \times 100 - mg \times 50 - 2mg \times 25 = 0$

$\Rightarrow T_A \times 100 = 100mg$

$\Rightarrow T_A = 1mg$ (Tension in the string at A)

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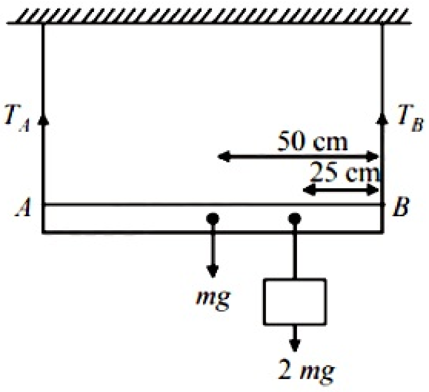
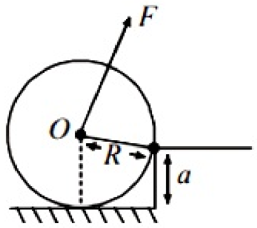


Question119

A uniform cylinder of mass M and radius R is to be pulled over a step of height a(a < R) by applying a force F at its centre ' O ' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :



[Sep. 02, 2020 (I)]

Options:

A. $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$

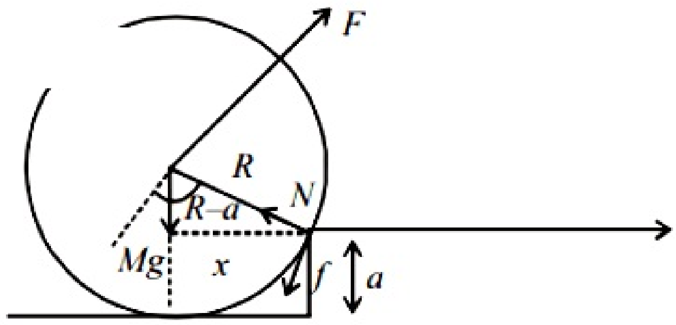
B. $Mg\sqrt{\left(\frac{R}{R-a}\right)^2-1}$

C. $Mg\frac{a}{R}$

D. $Mg\sqrt{1-\frac{a^2}{R^2}}$

Answer: A

Solution:



For step up, $F \times R \geq Mg \times x$

$x = \sqrt{R^2-(R-a)^2}$ from figure

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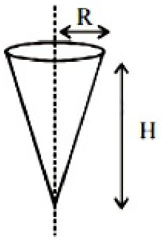
$$F_{min} = \frac{Mg}{R} \times \sqrt{R^2 - (R-a)^2} = Mg\sqrt{1 - \left(\frac{R-a}{R}\right)^2}$$

Question120

Shown in the figure is a hollow ice cream cone (it is open at the top). If its mass is M, radius of its top, R and height, H, then its moment of inertia about its axis is :



[Sep. 06, 2020 (I)]

Options:

A. $\frac{MR^2}{2}$

B. $\frac{M(R^2 + H^2)}{4}$

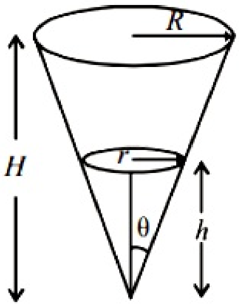
C. $\frac{MH^2}{3}$

D. $\frac{MR^2}{3}$

Answer: D

Solution:

Solution:
Hollow ice-cream cone can be assume as several parts of discs having different radius, so
$I = \int dI = \int dm(r^2)$ ......(i)



From diagram,
$\frac{r}{h} = \tan\theta = \frac{R}{H}$ or $r = \frac{R}{H}h$ ......(ii)
Mass of element, $dm = \rho(\pi r^2)dh$ ......(iii)
From eq. (i), (ii) and (iii),
Area of element, $dA = 2\pi r dl = 2\pi r\frac{dh}{\cos\theta}$
Mass of element, $dm = \frac{2Mh\tan d\ h}{R\sqrt{R^2 + H^2}\cos\theta}$
(here, $r = h\tan\theta$)
$I = \int dI = \int_0^H dm(r^2) = \int_0^H \rho(\pi r^2)dh\left(\frac{R}{H}.h\right)^2$

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$$= \int_0^H \rho\left(\pi\left(\frac{R}{H}.h\right)^4\right)dh$$

Solving we get, $I = \frac{MR^2}{2}$

Question121

The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1 + \frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod is:
[Sep. 06, 2020 (II)]

Options:

A. $\frac{5}{12}ML^2$

B. $\frac{7}{18}ML^2$

C. $\frac{2}{5}ML^2$

D. $\frac{3}{7}ML^2$

Answer: B

Solution:

Mass of the small element of the rod
$dm = \lambda . dx$
Moment of inertia of small element,

$$dI = dm . x^2 = \lambda_0\left(1 + \frac{x}{L}\right) . x^2 dx$$

Moment of inertia of the complete rod can be obtained by integration

$$I = \lambda_0 \int_0^L \left(x^2 + \frac{x^3}{L}\right)dx$$

$$= \lambda_0\left|\frac{x^3}{3} + \frac{x^4}{4L}\right|_0^L = \lambda_0\left[\frac{L^3}{3} + \frac{L^3}{4}\right]$$

$$\Rightarrow I = \frac{7\lambda_0 L^3}{12} \quad ....(i)$$

Mass of the thin rod,

$$M = \int_0^L \lambda dx = \int_0^L \lambda_0\left(1 + \frac{x}{L}\right)dx = \frac{3\lambda_0 L}{2}$$

$$\therefore \lambda_0 = \frac{2M}{3L}$$

$$\therefore I = \frac{7}{12}\left(\frac{2M}{3L}\right)L^3 \Rightarrow I = \frac{7}{18}ML^2$$

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Question122

A wheel is rotating freely with an angular speed w on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is :
[Sep. 05, 2020 (I)]

Options:

A. $\frac{5}{6}$

B. $\frac{1}{4}$

C. 0

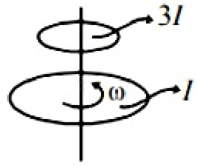
D. $\frac{3}{4}$

Answer: D

Solution:

Solution:
By angular momentum conservation, $L_c = L_f$

$\omega I + 3I \times 0 = 4I\omega' \Rightarrow \omega' = \frac{\omega}{4}$



$(KE)_i = \frac{1}{2}I\omega^2$

$(KE)_f = \frac{1}{2}(3I + I)\omega'^2$

$= \frac{1}{2} \times (4I) \times \left(\frac{\omega}{4}\right)^2 = \frac{I\omega^2}{8}$

$\Delta KE = \frac{1}{2}I\omega^2 - \frac{1}{8}I\omega^2 = \frac{3}{8}I\omega^2$

$\therefore$ Fractional loss in K . E . $= \frac{\Delta KE}{KE_{1_i}} = \frac{\frac{3}{8}I\omega^2}{\frac{1}{2}I\omega^2} = \frac{3}{4}$.

Question123

ABC is a plane lamina of the shape of an equilateral triangle. D, E are mid points of AB, AC and G is the centroid of the lamina. Moment of inertia of the lamina about an axis passing through G and perpendicular to the plane ABC is I_0 . If part ADE is removed, the moment of inertia of the remaining part about the same axis is $\frac{NI_0}{16}$ where N is an integer. Value of N is _______.

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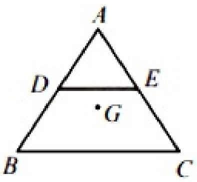
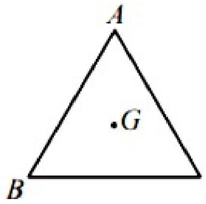


[NA Sep. 04, 2020 (I)]

Solution:

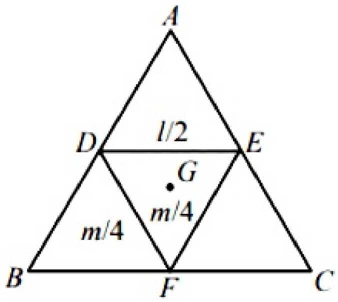
Let mass of triangular lamina $= m$, and length of side $= l$, then moment of inertia of lamina about an axis passing
oid G perpendicular to the plane.

$I_0 = kml^2$



Let moment of inertia of DE F $= I_1$ about G

So, $I_1 \propto \left(\frac{m}{4}\right)\left(\frac{l}{2}\right)^2 \propto \frac{ml^2}{16}$ or $I_1 = \frac{I_0}{16}$



Let $I_{ADE} = I_{BDF} = I_{EFC} = I_2$

$\therefore 3I_2 + I_1 = I_0 \Rightarrow 3I_2 + \frac{I_0}{16} = I_0 \Rightarrow I_2 = \frac{5I_0}{16}$

Hence, moment of inertia of DE CB i.e., after removal part ADE

$= 2I_2 + I_1 = 2\left(\frac{5I_0}{16}\right) + \left(\frac{I_0}{16}\right) = \frac{11I_0}{16} = \frac{NI_0}{16}$

Therefore value of $N = 11$

Question124

A circular disc of mass M and radius R is rotating about its axis with angular speed ω_1 . If another stationary dischaving radius $\frac{R}{2}$ and same mass M is dropped co-axially on to the rotating disc. Gradually both discs attain constant angular speed ω_1 . The energy lost in the process is p% of the initial energy. Value of p is __________.
[NA Sep. 04, 2020 (I)]

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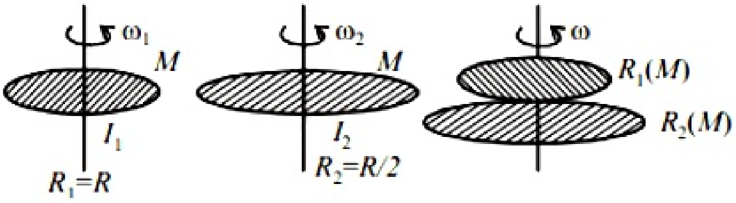
Answer: 20

Solution:

v moment of inertia disc, $I_{disc} = \frac{1}{2}MR^2$



Using angular momentum conservation

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2) \times \omega_f$

$\frac{MR^2}{2} \times \omega + 0 = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega_f \Rightarrow \omega_f = \frac{4}{5}\omega$

Initial K.E., $K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{MR^2}{2}\right)\omega^2 = \frac{MR^2\omega^2}{4}$

Final K.E., $K_f = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\frac{16}{25}\omega^2 = \frac{MR^2\omega^2}{5}$

Percentage loss in kinetic energy \% loss

$$= \frac{\frac{MR^2\omega^2}{4} - \frac{MR^2\omega^2}{5}}{\frac{MR^2\omega^2}{4}} \times 100 = 20\% = P\%$$

Hence, value of P = 20.

Question125

Consider two uniform discs of the same thickness and different radii $R_1 = R$ and $R_2 = \alpha R$ made of the same material. If the ratio of their moments of inertia I_1 and I_2, respectively, about their axes is $I_1 : I_2 = 1 : 16$ then the value of α is :
[Sep. 04, 2020 (II)]

Options:

A. $2\sqrt{2}$

B. $\sqrt{2}$

C. 2

D. 4

Answer: C

Solution:

Solution:
Let ρ be the density of the discs and t is the thickness of discs.
Moment of inertia of disc is given by

$I = \frac{MR^2}{2} = \frac{[\rho(\pi R^2)t]R^2}{2}$

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$I \propto R^4$(As ρ and t are same)

$\frac{I_2}{I_1} = \left(\frac{R_2}{R_1}\right)^4 \Rightarrow \frac{16}{1} = \alpha^4 \Rightarrow \alpha = 2$

Question126

For a uniform rectangular sheet shown in the figure, the ratio of moments of inertia about the axes perpendicular to the sheet and passing through O (the centre of mass) and O' (corner point) is : [Sep. 04, 2020 (II)]

Options:

A. 2/3

B. 1/4

C. 1/8

D. 1/2

Answer: B

Solution:

Solution:

Moment of inertia of rectangular sheet about an axis passing through O,

$I_O = \frac{M}{12}(a^2 + b^2) = \frac{M}{12}[(80)^2 + (60)^2]$

From the parallel axis theorem, moment of inertia about O′

$I_{O'} = I_O + M(50)^2$

$\frac{I_O}{I_{O'}} = \frac{\frac{M}{12}(80^2 + 60^2)}{\frac{M}{12}(80^2 + 60^2) + M(50)^2} = \frac{1}{4}$

Question127

Moment of inertia of a cylinder of mass M , length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4} + \frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L / R for it to have minimum possible I is: [Sep. 03, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{3}{2}$

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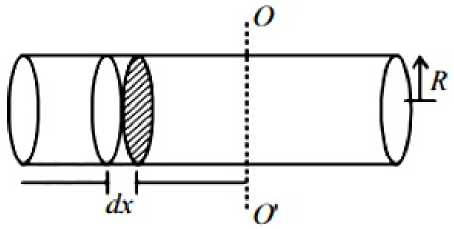
C. $\sqrt{\frac{3}{2}}$

D. $\sqrt{\frac{2}{3}}$

Answer: C

Solution:

Solution:
Let there be a cylinder of mass m length L and radius R. Now, take elementary disc of radius R and thickness dx at a distance of x from axis OO' then moment of inertia about OO' of this element.



$dI = \frac{dmR^2}{4} + dmx^2$

$\Rightarrow I = \int dI = \int \frac{dmR^2}{4} + \int_{n=L/2}^{n=-L/2} \frac{M}{L} dx \times x^2$

Given : $I = \frac{MR^2}{4} + \frac{ML^2}{12}$

$\Rightarrow I = \frac{M}{4} \times \frac{V}{\pi L} + \frac{ML^2}{12} \Rightarrow I = \frac{MV}{4\pi L} + \frac{ML^2}{12}$

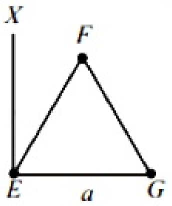
$\frac{dI}{dL} = -\frac{mV}{4\pi L^2} + \frac{M \times 2L}{12} = 0$

$\Rightarrow V = \frac{2}{3}\pi L^3 \Rightarrow \pi R^2 L = \frac{2}{3}\pi L^3$

$\therefore \frac{L}{R} = \sqrt{\frac{3}{2}}$

Question128

An massless equilateral triangle EFG of side ' a ' (As shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\frac{N}{20}ma^2$ where N is an integer. The value of N is ______.



[Sep. 03, 2020 (II)]

Answer: 25

Solution:

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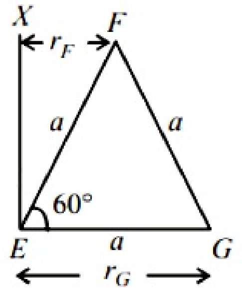
Moment of inertia of the system about axis XE.



$I = I_E + I_F + I_G$
$\Rightarrow I = m(r_E)^2 + m(r_F)^2 + m(r_G)^2$
$\Rightarrow I = m \times 0^2 + m\left(\frac{a}{2}\right)^2 + ma^2 = \frac{5}{4}ma^2 = \frac{25}{20}ma^2$
$\therefore N = 25$

Question129

Two uniform circular discs are rotating independently in the same direction around their common axis passing through their centres. The moment of inertia and angular velocity of the first disc are $0.1 kg - m^2$ and $10 rad\, s^{-1}$ respectively while those for the second one are $0.2 kg - m^2$ and $5 rad\, s^{-1}$ respectively. At some instant they get stuck together and start rotating as a single system about their common axis with some angular speed. The kinetic energy of the combined system is:
[Sep. 02,2020 (II)]

Options:

A. $\frac{10}{3}J$

B. $\frac{20}{3}J$

C. $\frac{5}{3}J$

D. $\frac{2}{3}J$

Answer: B

Solution:

Solution:
Initial angular momentum $= I_1\omega_1 + I_2\omega_2$
Let ω be angular speed of the combined system.
Final angular momentum $= I_1\omega + I_2\omega$
According to conservation of angular momentum
$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$
$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2} = \frac{0.1 \times 10 + 0.2 \times 5}{0.1 + 0.2} = \frac{20}{3}$
Final rotational kinetic energy
$K_f = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(0.1 + 0.2) \times \left(\frac{20}{3}\right)^2$
$\Rightarrow K_f = \frac{20}{3}J$

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Question130

A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be :
[12 Jan. 2019 II]

Options:

A. 2.0

B. 0.1

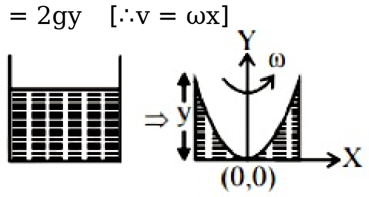
C. 0.4

D. 1.2

Answer: A

Solution:

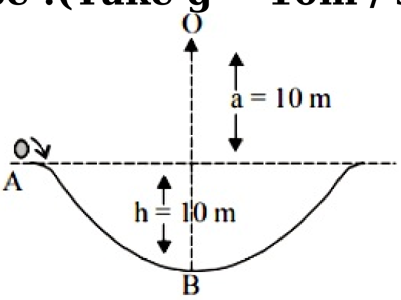
Solution:
Using $v^2 = u^2 + 2gy$ $[\because u = 0 \text{ at } (0, 0)]$
$v^2 = 2gy$ $[\because v = \omega x]$



$\Rightarrow y = \frac{\omega^2 x^2}{2g} = \frac{(2 \times 2\pi)^2 \times (0.05)^2}{20} \simeq 2\text{cm}$

Question131

A particle of mass 20g is released with an initial velocity 5m / s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be :(Take $g = 10m / s^2$)



[12 Jan. 2019 II]

Options:

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A. $2 kg - m^2 / s$

B. $8 kg - m^2 / s$

C. $6 kg - m^2 / s$

D. $3 kg - m^2 / s$

Answer: C

Solution:

According to work-energy theorem

$$\frac{1}{2}mv_B{}^2 - \frac{1}{2}mv_A{}^2$$

$$2gh = v_B{}^2 - v_A{}^2$$

$$2 \times 10 \times 10 = v_B{}^2 - 5^2$$

$$\Rightarrow v_B = 15 m / s$$

Angular momentum about O,

$$L_O = mvr$$

$$= 20 \times 10^{-3} \times 20$$

$$L_O = 6 kg . m^2 / s$$

Question132

The position vector of the centre of mass r_{cm} of an asymmetric uniform bar of negligible area of cross section as shown in figure is:

[12 Jan. 2019 I]

Options:

A. $\vec{r}cm = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

B. $\vec{r}cm = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

C. $\vec{r}cm = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

D. $\vec{r}cm = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

Answer: A

Solution:

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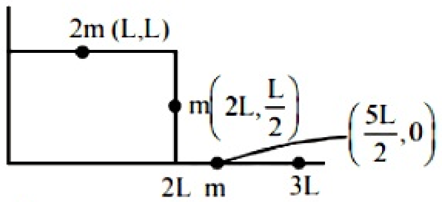
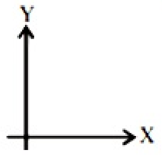




x-coordinate of centre of mass is

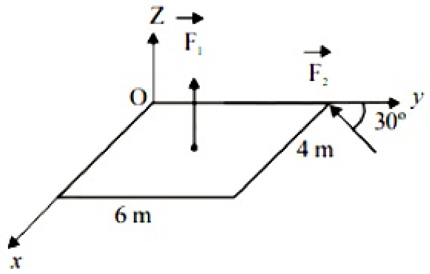
$$X_{cm} = \frac{2mL + 2mL + \frac{5mL}{2}}{4m} = \frac{13}{8}L$$

y-coordinate of centre of mass is

$$Y_{cm} = \frac{2m \times L + m \times \left(\frac{L}{2}\right) + m \times 0}{4m} = \frac{5L}{8}$$

Question133

A slab is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z - axis at the point $\left(2\vec{i} + 3\vec{j}\right)$. The moment of these forces about point O will be :



[11 Jan. 2019 I]

Options:

A. $\left(3\hat{i} - 2\hat{j} + 3\hat{k}\right)F$

B. $\left(3\hat{i} - 2\hat{j} - 3\hat{k}\right)F$

C. $\left(3\hat{i} + 2\hat{j} - 3\hat{k}\right)F$

D. $\left(3\hat{i} + 2\hat{j} + 3\hat{k}\right)F$

Answer: A

Solution:

` $= \frac{F}{2}\left(-\hat{i}\right) + \frac{F\sqrt{3}}{2}\left(-\hat{j}\right)$

$\vec{r} = 0\hat{i} + 6\hat{j}$

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Torque due to F_1 force

$\vec{\tau}_{F_1} = \vec{r}_1 \times \vec{F}_1 = 6\hat{j} \times \left(\frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})\right) = 3F(\hat{k})$

Torque due to F_2, force

$\vec{\tau}_{\vec{F}_2} = (2\hat{i} + 3\hat{j}) \times F\hat{k} = 3F\hat{i} + 2F(-\hat{j})$

$\vec{\tau}_{net} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} = 3F\hat{i} + 2F(-\hat{j}) + 3F(\hat{k})$

$= (3\hat{i} - 2\hat{j} + 3\hat{k})F$

Question134

The magnitude of torque on a particle of mass 1kg is 2.5 Nm about the origin. If the force acting on it is 1N , and the distance of the particle from the origin is 5m, the angle between the force and the position vector is (in radians):
[11 Jan. 2019 II]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{8}$

D. $\frac{\pi}{4}$

Answer: A

Solution:

Solution:

Torque about the origin $= \vec{\tau} = \vec{r} \times \vec{F}$

$= rF\sin\theta \Rightarrow 2.5 = 1 \times 5\sin\theta$

$\sin\theta = 0.5 = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Question135

To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is:
[10 Jan. 2019 I]

Options:

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A. $\mu F R / 3$

B. $\mu F R / 6$

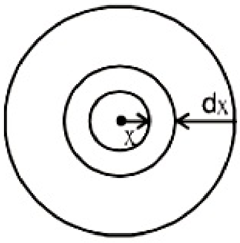
C. $\mu F R / 2$

D. $\frac{2}{3}\mu F R$

Answer: D

Solution:

Solution:



Consider a strip of radius x and thickness d x,
Torque due to friction on this strip
Net torque $= \sum$ Torque on ring

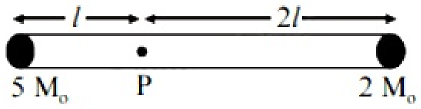
$\int d\tau = \int_0^R \frac{\mu F . 2\pi x d x}{\pi R^2}$

$\Rightarrow \tau = \frac{2\mu F}{R^2} \cdot \frac{R^3}{3}$

$\tau = \frac{2\mu F R}{3}$

Question136

A rigid massless rod of length 3l has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be:



[10 Jan. 2019 II]

Options:

A. $\frac{g}{13l}$

B. $\frac{g}{3l}$

C. $\frac{g}{2l}$

D. $\frac{7g}{3l}$

Answer: A

Solution:

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Applying torque equation about point P

$\tau = I\alpha = [2M_0(2l)^2 + 5M_0 l^2]\alpha$

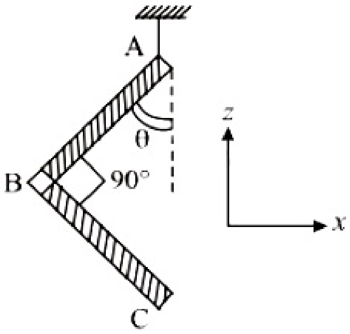
$\Rightarrow 5M_0 gl - 4M_0 gl = [2M_0(2l)^2 + 5M_0 l^2]\alpha$

$\Rightarrow M_0 gl = (13M_0 gl^2)\alpha$

$\therefore \alpha = \frac{g}{13l}$

Question137

An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is theta, then:



[9 Jan. 2019 I]

Options:

A. $\tan\theta = \frac{1}{2\sqrt{3}}$

B. $\tan\theta = \frac{1}{2}$

C. $\tan\theta = \frac{2}{\sqrt{3}}$

D. $\tan\theta = \frac{1}{3}$

Answer: D

Solution:

Solution:

Given that, the rod is of uniform mass density and AB = BC

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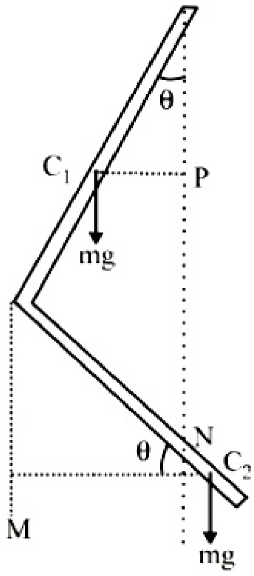

Let mass of one rod is m.
Balancing torque about hinge point.
$mg(C_1P) = mg(C_2N)$

$mg\left(\frac{L}{2}\sin\theta\right) = mg\left(\frac{L}{2}\cos\theta - L\sin\theta\right)$

$\Rightarrow \frac{3}{2}mgL\sin\theta = \frac{mgL}{2}\cos\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{3}$ or, $\tan\theta = \frac{1}{3}$

Question138

Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10cm and outer radius 20cm), about its axis be 1 . The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is:
[12 Jan. 2019 I]

Options:

A. 12cm

B. 16cm

C. 14cm

D. 18cm

Answer: B

Solution:

Solution:
Hint: Apply the formula for the moment of inertia of the hollow cylinder with inner and outer radius and find out the value of the moment of inertia of the hollow cylinder and then find out the moment of inertia of a thin cylinder with some assumed radius and equate the both of the moment of inertia to find the value of the assumed radius.
We know that the moment of inertia of a hollow cylinder with inner and outer radius about its axis can be written as

$I = m\left(\frac{R_i^2 + R_o^2}{2}\right)$

Where I is the moment of inertia of the cylinder.
The mass of the cylinder denotedby m
And the inner and outer radius are denoted by R_i and R_o

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Now we are given the inner radius of the cylinder as $10\mathrm{cm}$
The outer radius of the cylinder as $20\mathrm{cm}$
The moment of inertia of the hollow cylinder isgiven as I
So we can find the moment of inertia of the cylinder about its axis as

$$I = m\left(\frac{10^2 + 20^2}{2}\right)$$

Now we have found the moment of inertia of the hollow cylinder and let us know to find the moment of inertia of a thin cylinderwith no inner and outer radius.
We know the moment of inertia of thin cylinder as : $I = mk^2$
Where k is the radius of the cylinder and I is the moment of inertia of hollow cylinder
Now we are given that the moment of inertia of the two cylinders are equal and hence we can obtain an equation as below : $I = m\left(\frac{10^2 + 20^2}{2}\right) = mk^2$
Using the above equation we can get the value of the assumed radius of the hollow cylinder as

$$k = \sqrt{\frac{10^2 + 20^2}{2}}$$

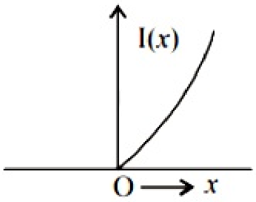
$k = 5\sqrt{10} = 16$ approx
Hence we have found the value of the radius of the cylinder as: $16\mathrm{cm}$

Question139

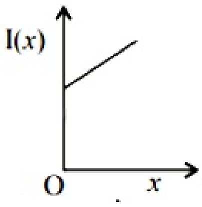
The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I (x) '. Which one of the graphs represents the variation of I (x) with x correctly? [12 Jan. 2019 II]

Options:

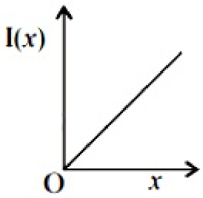
A.



B.



C.



D.

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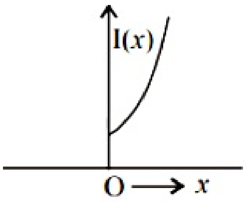


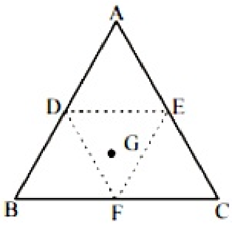
Answer: D

Solution:

Solution:
According to parallel axes theorem
$I = \frac{2}{5}mR^2 + mx^2$
Hence graph (d) correctly depicts I vs x.

Question140

An equilateral triangle ABC is cut from a thin solid sheet of wood. (See figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then:



[11 Jan. 2019 I]

Options:

A. $I = \frac{15}{16} I_0$

B. $I = \frac{3}{4} I_0$

C. $I = \frac{9}{16} I_0$

D. $I = \frac{I_0}{4}$

Answer: A

Solution:

Solution:
Let mass of the larger triangle $= M$
Side of larger triangle $= l$
Moment of inertia of larger triangle $= ma^2$

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Mass of smaller triangle $= \frac{M}{4}$

Length of smaller triangle $= \frac{l}{2}$

Moment of inertia of removed triangle $= \frac{M}{4}\left(\frac{a}{2}\right)^2$

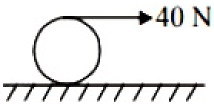
$\therefore \frac{I_{removed}}{I_{original}} = \frac{\frac{M}{4}}{M} \cdot \frac{\left(\frac{a}{2}\right)^2}{(a)^2}$

$I_{removed} = \frac{I_0}{16}$

So, $I = I_0 - \frac{I_0}{16} = \frac{15I_0}{16}$

Question141

a string is wound around a hollow cylinder of mass 5kg and radius 0.5m. If the string is now pulled with a horizontal force of 40N , and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string)



[11 Jan. 2019 II]

Options:

A. $20 rad / s^2$

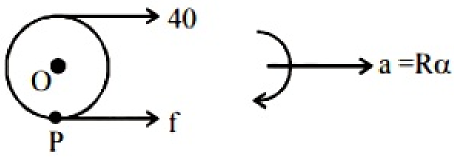
B. $16 rad / s^2$

C. $12 rad / s^2$

D. $10 rad / s^2$

Answer: B

Solution:



From newton's second law
$40 + f = m(R\alpha)$ ....(i)
Taking torque about 0 we get
$40 \times R - f \times R = I\alpha$
$40 \times R - f \times R = mR^2\alpha$
$40 - f = mR\alpha$ ....(ii)
Solving equation (i) and (ii)
$\alpha = \frac{40}{mR} = 16 rad / s^2$

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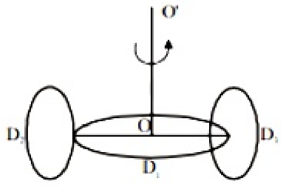
Question142

A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be :



[11 Jan. 2019 II]

Options:

A. MR^2

B. $3MR^2$

C. $\frac{4}{5}MR^2$

D. $\frac{2}{3}MR^2$

Answer: B

Solution:

Solution:

Moment of inertia of disc D_1 about $OO' = I_1 = \frac{MR^2}{2}$

M.O.I of D_2 about OO'

$= I_2 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$

M.O.I of D_3 about OO'

$= I_3 = \frac{1}{2}\left(\frac{MR^2}{2}\right) + MR^2 = \frac{MR^2}{4} + MR^2$

so, resultant M.O.I about OO' is $I = I_1 + I_2 + I_3$

$\Rightarrow I = \frac{MR^2}{2} + 2\left(\frac{MR^2}{4} + MR^2\right)$

$= \frac{MR^2}{2} + \frac{MR^2}{2} + 2MR^2 = 3MR^2$

Question143

Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is:

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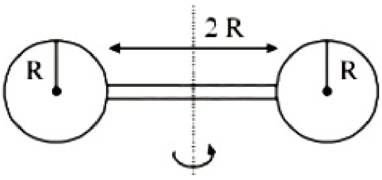


[10 Jan. 2019 II]

Options:

A. $\frac{137}{15}MR^2$

B. $\frac{17}{15}MR^2$

C. $\frac{209}{15}MR^2$

D. $\frac{152}{15}MR^2$

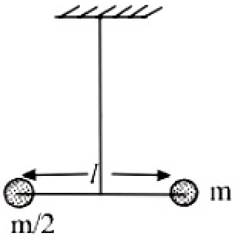
Answer: A

Solution:

Solution:
For Ball
using parallel axes theorem, for ball moment of inertaia,
$I_{ball} = \frac{2}{5}MR^2 + M(2R)^2 = \frac{22}{5}MR^2$
For two balls $I_{balls} = 2 \times \frac{22}{5}MR^2 =$ and,
$I_{rod} = \frac{M(2R)^2}{12} = \frac{MR^2}{3}$
$I_{system} = I_{balls} + I_{rod}$
$= \frac{44}{5}MR^2 + \frac{MR^2}{3} = \frac{137}{15}MR^2$

Question144

Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring toruque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:



[9 Jan. 2019 I]

Options:

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A. $\frac{3k\theta_0^2}{l}$

B. $2k\theta_0^2 l$

C. $\frac{k\theta_0^2}{l}$

D. $\frac{k\theta_0^2}{2l}$

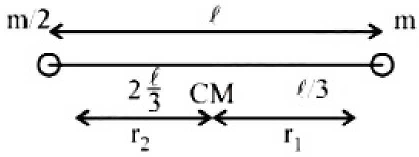
Answer: C

Solution:

Solution:

As we know, $\omega = \sqrt{\frac{k}{I}}$

$\omega = \sqrt{\frac{3k}{ml^2}} \left[\because I_{rod} = \frac{1}{3}ml^2\right]$

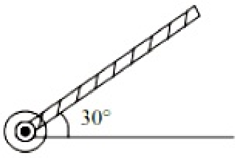


Tension when it passes through the mean position,

$= m\omega^2\theta_0^2\frac{l}{3} = m\frac{3k}{ml^2}\theta_0^2\frac{l}{3} = \frac{k\theta_0^2}{l}$

Question145

A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rads $^{-1}$) will be ($g = 10ms^{-2}$)



[9 Jan. 2019 II]

Options:

A. $\sqrt{\frac{30}{7}}$

B. $\sqrt{30}$

C. $\sqrt{\frac{20}{3}}$

D. $\sqrt{\frac{30}{2}}$

Answer: D

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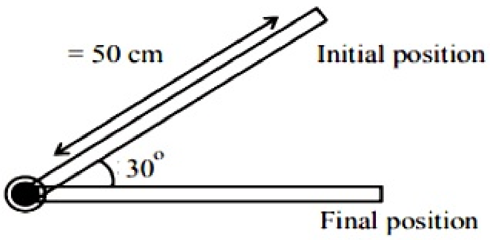
Solution:

Solution:



By the low of conservation of energy,
P.E. of rod = Rotational K . E .

$mg\frac{l}{2}\sin\theta = \frac{1}{2}I\omega^2$

$\Rightarrow mg\frac{l}{2}\sin 30° = \frac{1}{2}\frac{ml^2}{3}\omega^2 \Rightarrow mg\frac{l}{2} \times \frac{1}{2} = \frac{1}{2}\frac{ml^2}{3}\omega^2$

For complete length of rod,

$\omega = \sqrt{3g/2(2l)} = \sqrt{\frac{30}{2}}\text{ rod s}^{-1}$

Question146

A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is:
[10 Jan. 2019 I]

Options:

A. $\frac{3F}{2mR}$

B. $\frac{F}{3mR}$

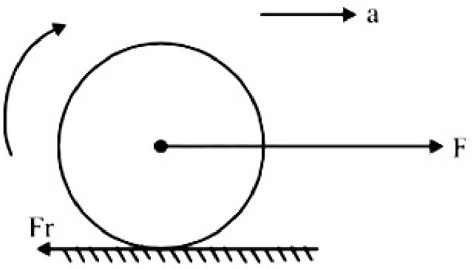
C. $\frac{F}{2mR}$

D. $\frac{2F}{3mR}$

Answer: D

Solution:

Solution:



$F - fr = ma$ .....(I)

$f_r R = I\alpha = \frac{mR^2}{2}\alpha$ .....(ii)

for pure rolling

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$a = \alpha R$ ......(iii)
from (1)(2) and (3)

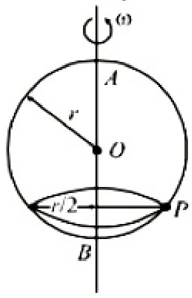
$F - \frac{mR\alpha}{2} = m\alpha R$

$F = \frac{3}{2}mR\alpha$

$\alpha = \frac{2F}{3mR}$

Question147

A smooth wire of length 2πr is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to :



[12 Apr. 2019 II]

Options:

A. $\frac{\sqrt{3}g}{2r}$

B. $2g / (r\sqrt{3})$

C. $(g\sqrt{3}) / r$

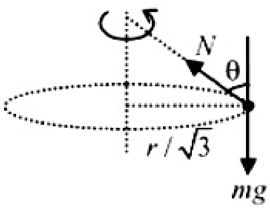
D. $2g / r$

Answer: B

Solution:

Solution:

$N \sin\theta = m\omega^2(r / 2)$ ......(i)



$\sin\theta = \frac{r / 2}{r} = \frac{1}{2} \Rightarrow \theta = 30°$

and $N \cos\theta = mg$ .....(ii)

or $\tan\theta = \frac{\omega^2 r}{2g}$

or $\tan 30° = \frac{\omega^2 r}{2g}$

or $\frac{1}{\sqrt{3}} = \frac{\omega^2 r}{2g}$

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$\therefore \omega^2 = \frac{2g}{r\sqrt{3}}$

Question148

A particle of mass m is moving along a trajectory given by
$x = x_0 + a\cos\omega_1 t$
$y = y_0 + b\cos\omega_2 t$
The torque, acting on the particle about the origin, at t = 0 is:
[10 Apr. 2019 I]

Options:

A. $m(-x_0 b + y_0 a)\omega_1^{\ 2}\hat{k}$

B. $+m y_0 a \omega_1^{\ 2}\hat{k}$

C. zero

D. $-m(x_0 b\omega_2^{\ 2} - y_0 a\omega_1^{\ 2})\hat{k}$

Answer: B

Solution:

Solution:
Given that, $x = x_0 + a\cos\omega_1 t$
$y = y_0 + b\sin\omega_2 t$
$\frac{dx}{dt} = V_x$
$\Rightarrow v_x = -a\omega_1\sin(\omega_1 t)$, and $\frac{dy}{dt} = v_y = b\omega_2\cos(\omega_2 t)$
$\frac{dv_x}{dt} = a_x = -a\omega_1^{\ 2}\cos(\omega_1 t)$, $\frac{dv_y}{dt} = a_y = -b\omega_2^{\ 2}\sin(\omega_2 t)$
At $t = 0$, $x = x_0 + a$, $y = y_0$
$a_x = -a\omega_1^{\ 2}$, $a_y = 0$
Now, $\vec{\tau} = \vec{r}\times\vec{F} = m(\vec{r}\times\vec{a})$
$= \left[(x_0 + a)\hat{i} + y_0\hat{j}\right]\times m\left(-a\omega_1^{\ 2}\hat{i}\right) = +m y_0 a\omega_1^{\ 2}\hat{k}$

Question149

The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is :
[10 Apr. 2019 II]

Options:

A. $48\left(\hat{i} + \hat{j}\right)$

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B. $36\hat{k}$

C. $-34(\hat{k}-\hat{i})$

D. $-48\hat{k}$

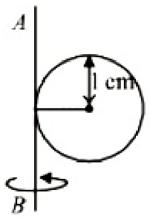
Answer: D

Solution:

Solution:
We have given $\vec{r} = 2t\hat{i} - 3t^2\hat{j}$
$\vec{r}(\text{ at } t = 2) = 4\hat{i} - 12\hat{j}$
Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} - 6\hat{t}$
$\vec{v}(\text{ at } t = 2) = 2\hat{i} - 12\hat{j}$
$\vec{L} = mvr\sin\theta\,\hat{n} = m(\vec{r} \times \vec{v})$
$= 2(4\hat{i} - 12\hat{j}) \times (2\hat{i} - 12\hat{j}) = -48\hat{k}$

Question150

A metal coin of mass 5g and radius 1cm is fixed to a thin stick AB of negligible mass as shown in the figure The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to :



[10 Apr. 2019 II]

Options:

A. 4.0×10^{-6} N m

B. 1.6×10^{-5} N m

C. 7.9×10^{-6} N m

D. 2.0×10^{-5} N m

Answer: D

Solution:

Solution:
Angular acceleration,
$\alpha = \frac{\omega - \omega_0}{t} = \frac{25 \times 2\pi - 0}{5} = 10\pi\text{rad} / s^2$
$\tau = I\alpha$

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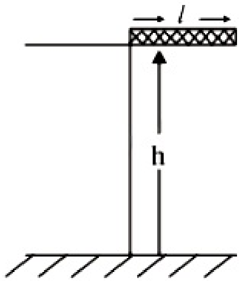
$\Rightarrow \tau = \left(\frac{5}{4}mR^2\right)\alpha \approx \left(\frac{5}{4}\right)(5 \times 10^{-3})(10^{-4})10\pi$

$= 2.0 \times 10^{-5} N\,m$

Question151

A rectangular solid box of length 0.3m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time τ = 0.01s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :



[8 Apr. 2019 II]

Options:

A. 0.5

B. 0.3

C. 0.02

D. 0.28

Answer: A

Solution:

Solution:
Angular impulse = change in angular momentum
$\tau \Delta t = \Delta L$

$mg\frac{\ell}{2} \times .01 = \frac{m\ell^2}{3}\omega$

$\omega = \frac{3g \times 0.01}{2\ell}$

$= \frac{3 \times 10 \times .01}{2 \times 0.3}$

$= \frac{1}{2} = 0.5 rad / s$

time taken by rod to hit the ground

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1 sec$

in this time angle rotate by rod
$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

Question152

A circular disc of radius b has a hole of radius a at its centre (see

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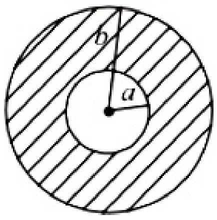
figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is:



[12 Apr. 2019 I]

Options:

A. $\sqrt{\frac{a^2+b^2+ab}{2}}$

B. $\frac{a+b}{2}$

C. $\sqrt{\frac{a^2+b^2+ab}{3}}$

D. $\frac{a+b}{3}$

Answer: C

Solution:

Solution:

$I = \int_a^b (dm) r^2$

$= \int_a^b \left(\frac{\sigma_0}{r} \times 2\pi r dr\right) r^2 = \frac{2\pi\sigma_0}{3} |r^3|_a^b$

$= \frac{2\pi\sigma_0}{3}(b^3 - a^3)$

Mass of the disc,

$m = \int_a^b \frac{\sigma_0}{r} \times 2\pi r dr = 2\pi\sigma_0(b-a)$

Radius of gyration,

$k = \sqrt{\frac{I}{m}}$

$= \sqrt{\frac{(2\pi\sigma_0/3)(b^3-a^3)}{2\pi\sigma_0(b-a)}} = \sqrt{\frac{a^2+b^2+ab}{3}}$

Question153

Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is:

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[10 Apr. 2019 I]

Options:

A. $-\frac{I_1\omega_1^2}{12}$

B. $\frac{I_1\omega_1^2}{6}$

C. $\frac{3}{8}I_1\omega_1^2$

D. $-\frac{I_1\omega_1^2}{24}$

Answer: D

Solution:

Solution:
As no external torque is acting so angular momentum should be conserved
$(I_1 + I_2)\omega = I_1\omega_1 + I_2\omega_2$[ω_c = . common angular velocity of the system, whendiscs are in contact]

$$\omega_c = \frac{I_1\omega_1 \frac{I_1\omega_1}{4}}{I_1 + \frac{I_1}{2}}\left(\frac{5}{4} \times \frac{2}{3}\right)\omega_1$$

$$\omega_c = \frac{5\omega_1}{6}$$

$$E_f - E_i = \frac{1}{2}(I_1 + I_2)\omega_c^2 - \frac{1}{2}I_1\omega_1^2 - \frac{1}{2}I_2\omega_2^2$$

Put $I_2 = I_1 / 2$and $\omega_c = \frac{5\omega_1}{6}5\omega_1 / 6$

We get :

$$E_f - E_i = -\frac{I_1\omega_1^2}{24}$$

Question154

A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is: [10 Apr. 2019 I]

Options:

A. $\frac{MR^2}{3}$

B. $\frac{2MR^2}{3}$

C. $\frac{MR^2}{6}$

D. $\frac{MR^2}{2}$

Answer: B

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Solution:

Solution:

As from the question density $(\sigma) = kr^2$

Mass of disc $M = \int_0^R (kr^2) 2\pi r d r = 2\pi k \frac{R^4}{4} = \frac{\pi k R^4}{2}$

$\Rightarrow k = 2\frac{M}{\pi R^4}$ ....(i)

∴ Moment of inertia about the axis of the disc.

$I = \int d I = \int (d m) r^2 = \int \sigma d A r^2$

$= \int (kr^2)(2\pi r d r) r^2$

$= \int_0^R 2\pi k r^5 d r = \frac{\pi k R^6}{3} = \frac{\pi \times \left(\frac{2M}{\pi R^4}\right) \times R^6}{3} = \frac{2}{3} M R^2$ [putting value of k from eqn .... (i)]

Question155

A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the new sphere about its axis. The ratio I_1 / I_2 is given by:
[10 Apr. 2019 II]

Options:

A. 185

B. 140

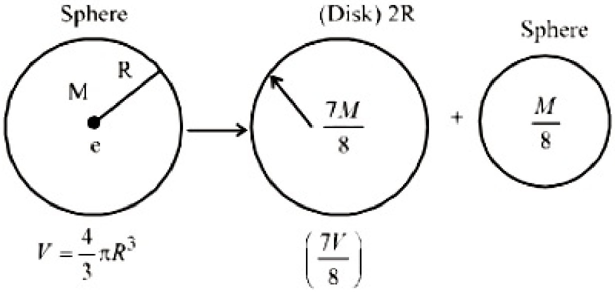
C. 285

D. 65

Answer: B

Solution:

Solution:



$I_1 = \left(\frac{7M}{8}\right)(2R)^2 \frac{1}{2} = \left(\frac{7}{16} \times 4\right) M R^2 = \frac{14}{8} m R^2$

$I_2 = \frac{2}{5}\left(\frac{M}{8}\right) r^2$

$\Rightarrow I_2 = \frac{2}{5}\left(\frac{M}{8}\right)\left(\frac{R^2}{4}\right) = \frac{M R^2}{80}$

$\left[\frac{4}{3}\pi r^3 \rho = \frac{1}{8}\frac{4}{3}\pi R^3 \times \rho \Rightarrow r = \frac{R}{2}\right]$

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$\frac{I_1}{I_2} = \frac{14 \times 80}{8} = 140$

Question156

A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is:
[9 April 2019 I]

Options:

A. $\frac{k}{4I}\theta$

B. $\frac{k}{I}\theta$

C. $\frac{k}{2I}\theta$

D. $\frac{2k}{I}\theta$

Answer: D

Solution:

Solution:

$\frac{1}{2}I\omega^2 = kQ^2$

or $\omega = \left(\sqrt{\frac{2k}{I}}\right)Q$

or $\alpha = \frac{d\omega}{\pi} = \sqrt{\frac{2K}{I}}\left(\frac{dQ}{dt}\right) = \left(\sqrt{\frac{2k}{I}}\right)\omega$

$= \left(\sqrt{\frac{2k}{I}}\right)\left(\sqrt{\frac{2k}{I}}\right)\theta = \frac{2k\theta}{I}$

Question157

Moment of inertia of a body about a given axis is $1.5 kg\ m^2$. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200J , the angular acceleration of $20 rad\ / s^2$ must be applied about the axis for a duration of:
[9 Apr. 2019 II]

Options:

A. 2.5s

B. 2s

C. 5s

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D. 3s

Answer: B

Solution:

Solution:
$\omega = \alpha t = 20t$
Given, $\frac{1}{2} I \omega^2 = 1200$
or $\frac{1}{2} \times 1.5 \times (20t)^2 = 1200$
or $t = 2s$

Question158

A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be:
[9 Apr. 2019 II]

Options:

A. $\frac{M\omega_0}{M + m}$

B. $\frac{M\omega_0}{M + 3m}$

C. $\frac{M\omega_0}{M + 6m}$

D. $\frac{M\omega_0}{M + 2m}$

Answer: C

Solution:

Solution:
$I_i\omega_i = I_f\omega_f$
or $\left(\frac{ML^2}{12}\right)\omega_0 = \left(\frac{ML^2}{12} + 2m\left(\frac{L}{2}\right)^2\right)\omega_f$
$: \omega_f = \left(\frac{M\omega_0}{M + 6m}\right)$

Question159

A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its center. The

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moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is:
[8 April 2019 I]

Options:

A. 1 / 2

B. 3 / 5

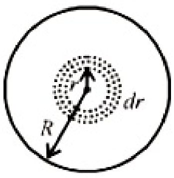
C. 8 / 5

D. 3 / 2

Answer: C

Solution:

Solution:
Taking a circular ring of radius r and thickness dr as a mass element, so total mass,



$$M = \int_0^R \rho_0 r \times 2\pi r dr = \frac{2\pi\rho_0 R^3}{3}$$

$$I_C = \int_0^R \rho_0 r \times 2\pi r dr \times r^2 = \frac{2\pi\rho_0 R^5}{5}$$

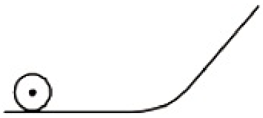
Using parallel axis theorem

$$\therefore I = I_C + MR^2 = 2\pi\rho_0 R^5\left(\frac{1}{3}+\frac{1}{5}\right) = \frac{16\pi\rho_0 R^5}{15}$$

$$= \frac{8}{5}\left[\frac{2}{3}\pi\rho_0 R^3\right]R^2 = \frac{8}{5}MR^2$$

Question160

A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by:



[8 Apr. 2019 II]

Options:

A. $\frac{2}{\sqrt{5}}$

B. 1

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C. $\frac{14}{15}$

D. $\frac{4}{5}$

Answer: C

Solution:

Solution:
For sphere,

$\frac{1}{2}mv^2 + I\omega^2 = \frac{1}{2}mgh$

or $\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{v^2}{R^2} = mgh$ or $h = \frac{7v^2}{10g}$

For cylinder

$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right) = mgh'$

or $h' = \frac{3v^2}{4g}$

$\therefore \frac{h}{h'} = \frac{7v^2/10g}{3v^2/4g} = \frac{14}{15}$

Question161

The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane:
(i) a ring of radius R(ii) a solid cylinder of radius $\frac{R}{2}$and (iii) a solid sphere of radius $\frac{R}{4}$. If, in each case, thespeed of the center of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is:
[9 April 2019 I]

Options:

A. 4: 3: 2

B. 10: 15: 7

C. 20: 15: 14

D. 2: 3: 4

Answer: C

Solution:

Solution:

$mgh = \frac{1}{2}mv_{cm}{}^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{1}{2}mv_{cm}{}^2 + \frac{1}{2}I_{cm}\left(\frac{v_{cm}}{R}\right)^2$

$= \frac{1}{2}\left(m + \frac{I_{cm}}{R^2}\right)v_{cm}{}^2$

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For ring :$mgh = \frac{1}{2}\left(m + \frac{mR^2}{R^2}\right){v_{cm}}^2$

$\therefore h = \frac{{v_{cm}}^2}{g}$.

For solid cylinder, $mgh = \frac{1}{2}\left(m + \frac{mR^2}{2R^2}\right){v_{cm}}^2$

$\therefore h = \frac{3{v_{cm}}^2}{4g}$

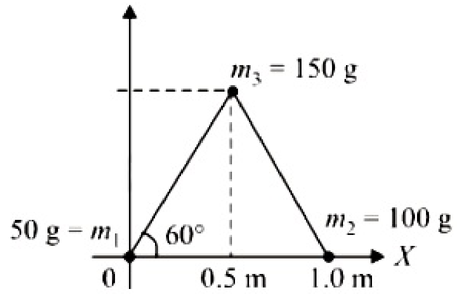
For sphere, $mgh = \frac{1}{2}\left(m + \frac{2}{5}\frac{mR^2}{R^2}\right){v_{cm}}^2$

$\therefore h = \frac{7{v_{cm}}^2}{10g}$

Ratio of heights $1 : \frac{3}{4} : \frac{7}{10} \Rightarrow 20 : 15 : 14$

Question162

Three particles of masses 50g, 100g and 150g are placed at the vertices of an equilateral triangle of side 1m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :



[12 Apr. 2019 II]

Options:

A. $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$

B. $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$

C. $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$

D. $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

Answer: C

Solution:

Solution:

$x_{cm} = \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150} = \frac{7}{12}m$

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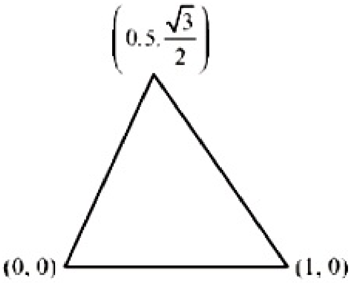
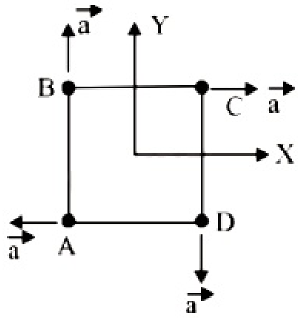


$$y_{cm} = \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150} = \frac{\sqrt{3}}{4} m$$

Question163

Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is :



[8 April 2019 I]

Options:

A. $\frac{a}{5}\left(\hat{i} - \hat{j}\right)$

B. a

C. Zero

D. $\frac{a}{5}\left(\hat{i} + \hat{j}\right)$

Answer: A

Solution:

Solution:

Acceleration of centre of mass (a_{cm} right) is given by $\therefore \vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots..}{m_1 + m_2 + \ldots\ldots..}$

$$= \frac{(2m)a\hat{j} + 3m \times a\hat{i} + ma\left(-\hat{i}\right) + 4m \times a\left(-\hat{j}\right)}{2m + 3m + 4m + m}$$

$$= \frac{2a\hat{i} - 2a\hat{j}}{10} = \frac{a}{5}\left(\hat{i} - \hat{j}\right)$$

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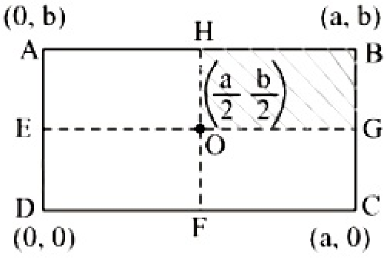
Question164

A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :



[8 Apr. 2019 II]

Options:

A. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

B. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

C. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

D. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

Answer: D

Solution:

Solution:

With respect to point θ, the CM of the cut-off portion $\left(\frac{a}{4}, \frac{b}{4}\right)$.

Using, $x_{CM} = \frac{MX - mx}{M - m}$

$$= \frac{M \times 0 - \frac{M}{4} \times \frac{a}{4}}{M - \frac{M}{4}} = -\frac{a}{12}$$

and $y_{CM} = -\frac{b}{12}$

So CM coordinates one

$x_0 = \frac{a}{2} - \frac{a}{12} = \frac{5a}{12}$

and $y_0 = \frac{b}{2} - \frac{b}{12} = \frac{5b}{12}$

Question165

A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is T (x) at a distance x from the axis, then which of the following graphs depicts it most closely?

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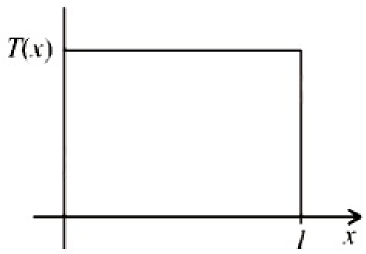
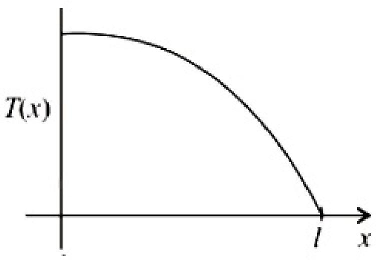
[12 Apr. 2019 II]

Options:

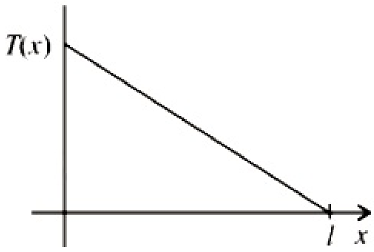
A.



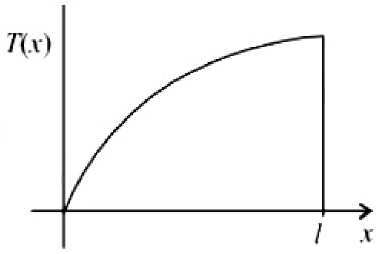
B.



C.



D.

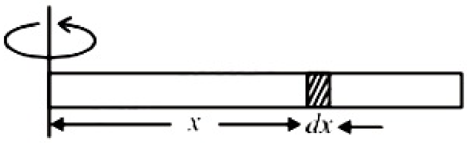


Answer: D

Solution:

Solution:

$\int_0^T (-dT) = \int_l^x (dm)\omega^2 x$



$-T = \int_l^x \left(\frac{m}{l} dx\right) \omega^2 x$

or $T = \frac{m\omega^2}{l}(l^2 - x^2)$

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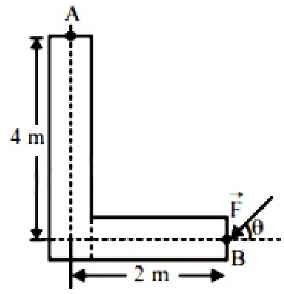
It is a parabola between T and x.

Question166

A force of 40N acts on a point B at the end of an L -shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by:



[Online April 15, 2018]

Options:

A. $\tan\theta = \frac{1}{4}$

B. $\tan\theta = 2$

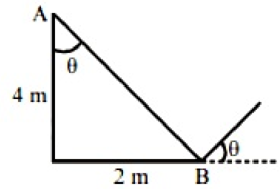
C. $\tan\theta = \frac{1}{2}$

D. $\tan\theta = 4$

Answer: C

Solution:

Solution:
To produce maximum moment of force line of action of force must be perpendicular to line AB.



$\therefore \tan\theta = \frac{2}{4} = \frac{1}{2}$

Question167

A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then:
[2018]

Options:

A. $T \propto R^{3/2}$ for any n.

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B. T $\propto R^{n/2+1}$

C. T $\propto R^{(n+1)/2}$

D. T $\propto R^{n/2}$

Answer: C

Solution:

Solution:

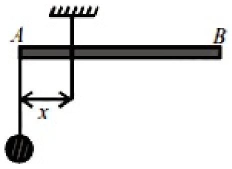
$m\omega^2 R = \text{ Force } \propto \frac{1}{R^n}\left(\text{ Force } = \frac{mv^2}{R}\right)$

$\Rightarrow \omega^2 \propto \frac{1}{R^{n+1}} \Rightarrow \omega \propto \frac{1}{R^{\frac{n+1}{2}}}$

Time period $T = \frac{2\pi}{\omega}$

Time period, $T \propto R^{\frac{n+1}{2}}$

Question168



A uniform rod AB is suspended from a point X , at a variable distance from x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variable that give a straight line, when plotted, are: [Online April 15, 2018]

Options:

A. m, $\frac{1}{x}$

B. m, $\frac{1}{x^2}$

C. m, x

D. m, x^2

Answer: A

Solution:

Solution:

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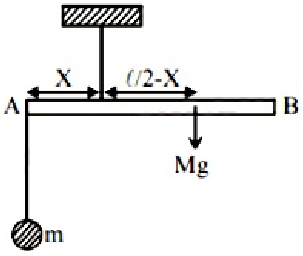


Balancing torque w.r.t. point of suspension

$mgx = M\,g\left(\frac{l}{2} - x\right)$

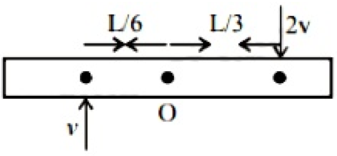
$\Rightarrow mx = M\frac{l}{2} - M\,x$

$m = \left(M\frac{l}{2}\right)\frac{1}{x} - M$

$y = \alpha\frac{1}{x} - C$ Straight line equation.

Question169

A thin uniform bar of length L and mass 8m lies on a smooth horizontal table. Two point masses m and 2m moving in the same horizontal plane from opposite sides of the bar with speeds 2v and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be:



[Online April 15, 2018]

Options:

A. $\frac{v}{6L}$

B. $\frac{6v}{5L}$

C. $\frac{3v}{5L}$

D. $\frac{v}{5L}$

Answer: A

Solution:

Solution:

Question170

Seven identical circular planar disks, each of mass M and radius R are

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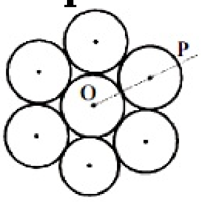
welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



[2018]

Options:

A. $\frac{19}{2}MR^2$

B. $\frac{55}{2}MR^2$

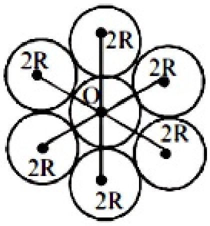
C. $\frac{73}{2}MR^2$

D. $\frac{181}{2}MR^2$

Answer: D

Solution:

Solution:



Using parallel axes theorem, moment of inertia about ' O '

$I_o = I_{cm} + md^2$

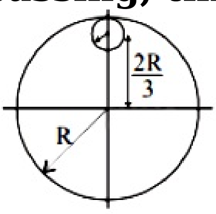
$= 7MR^22 + 6(M \times (2R)^2) = \frac{55MR^2}{2}$

Again, moment of inertia about point P, $I_p = I_o + md^2$

$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$

Question171

From a uniform circular disc of radius R and mass 9M , asmall disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing, through centre of disc is :



[2018]

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Options:

A. $4MR^2$

B. $\frac{40}{9}MR^2$

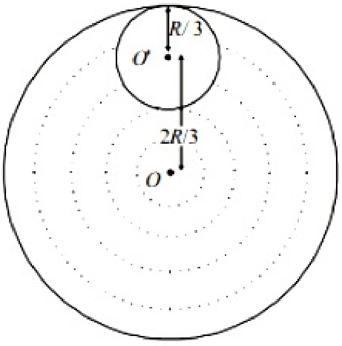
C. $10MR^2$

D. $\frac{37}{9}MR^2$

Answer: A

Solution:

Solution:
Let σ be the mass per unit area.



The total mass of the disc $= \sigma \times \pi R^2 = 9M$

The mass of the circular disc cut $= \sigma \times \pi\left(\frac{R}{3}\right)^2 = \sigma \times \frac{\pi R^2}{9} = M$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it.

Moment of inertia (I_1) of the complete disc $= \frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

$M.I.$ of the cut out portion about an axis passing through O′ and perpendicular to the plane of disc $= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$

∴ M.I. (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc $= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$

∴ M.I. (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$[Using perpendicular axis theorem]

∴ The total $M.I.$ of the system about an axis passing through O and perpendicular to the plane of the disc is

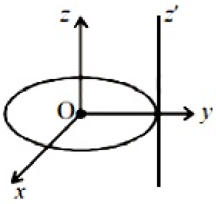
$I = I_1 + I_2$

$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$

$= \frac{9MR^2}{2} - \frac{9MR^2}{18} = \frac{(9-1)MR^2}{2} = 4MR^2$

Question172

A thin circular disk is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z′ axes will be



[Online April 16, 2018]

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Options:

A. 1: 2

B. 1: 4

C. 1: 3

D. 1: 5

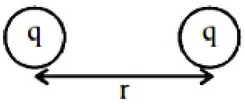
Answer: C

Solution:

Solution:
As we know, moment of inertia of a disc about an axis passing through C.G. and perpendicular to its plane,

$I_z = \frac{mR^2}{2}$

Moment of inertia of a disc about a tangential axis perpendicular to its own plane,

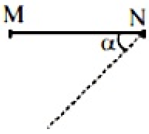


$I_z' = \frac{3}{2}mR^2$

$\therefore I_z / I_z' = \frac{mR^2}{2} / \frac{3mR^2}{2} = 1/3$

Question173

A thin rod M N , free to rotate in the vertical plane about the fixed end N , is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to: (see figure)



[Online April 15, 2018]

Options:

A. $\sqrt{\cos\alpha}$

B. $\cos\alpha$

C. $\sin\alpha$

D. $\sqrt{\sin\alpha}$

Answer: A

Solution:

Solution:
When the rod makes an angle α

Displacement of centre of mass $= \frac{l}{2}\cos\alpha$

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$mg\frac{l}{2}\cos\alpha = \frac{1}{2}I\,\omega^2$

$mg\frac{l}{2}\cos\alpha = \frac{ml^2}{6}\omega^2$ ($\because$ M.I. of thin uniform rod about an axis passing through its centre of mass and perpendicular to the rod $I = \frac{ml^2}{12}$)

$\Rightarrow \omega = \sqrt{\frac{3g\cos\alpha}{l}}$

Speed of end $= \omega \times l = \sqrt{3g\cos\alpha\, l}$

i.e., Speed of end, $\omega \propto \sqrt{\cos\alpha}$

Question174

In a physical balance working on the principle of moments, when 5mg weight is placed on the left pan, the beam becomes horizontal. Both the empty pans of the balance are of equal mass. Which of the following statements is correct?
[Online April 8, 2017]

Options:

A. Left arm is longer than the right arm

B. Both the arms are of same length

C. Left arm is shorter than the right arm

D. Every object that is weighed using this balance appears lighter than its actual weight.

Answer: C

Solution:

Solution:
According to principle of moments when a system is stable or balance, the anti-clockwise moment is equal to clockwise moment.
i.e., load × load arm = effort × effort arm
When $5mg$ weight is placed, load arm shifts to left side, hence left arm becomes shorter than right arm.

Question175

The machine as shown has 2 rods of length 1m connected by a pivot at the top. The end of one rod is connected to the floor by a stationary pivot and the end of the other rod has a roller that rolls along the floor in a slot.
As the roller goes back and forth, a 2kg weight moves up and down. If the roller is moving towards right at a constant speed, the weight moves up with a :

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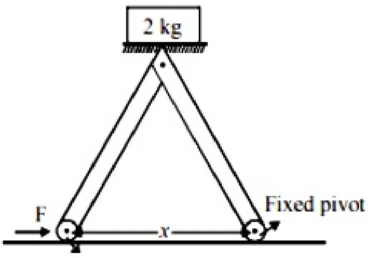


[Online April 9, 2017]

Options:

A. constant speed

B. decreasing speed

C. increasing speed

D. speed which is $\frac{3}{4}$ th of that of the roller when the weight is 0.4m above the ground

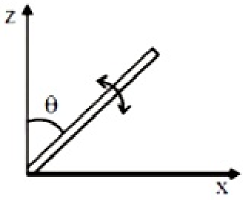
Answer: B

Solution:

Solution:
decreasing speed

Question176

A slender uniform rod of mass M and length ell is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



[2017]

Options:

A. $\frac{3g}{2l}\cos\theta$

B. $\frac{2g}{3l}\cos\theta$

C. $\frac{3g}{2l}\sin\theta$

D. $\frac{2g}{2l}\sin\theta$

Answer: C

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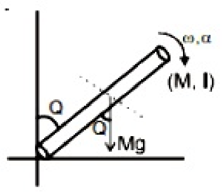
Solution:

Solution:



Torque at angle θ

$\tau = M g \sin\theta \cdot \frac{l}{2}$

Also $\tau = I\alpha$

$\therefore I\alpha = M g \sin\theta \frac{l}{2}$

$\frac{M l^2}{3} \cdot \alpha = M g \sin\theta \frac{l}{2} \left[\because I_{rod} = \frac{M l^2}{3} \right]$

$\Rightarrow \frac{l\alpha}{3} = g\frac{\sin\theta}{2} \therefore \alpha = \frac{3g\sin\theta}{2l}$

Question177

The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I . What is the ratio l / R such that the moment of inertia is minimum?
[2017]

Options:

A. 1

B. $\frac{3}{\sqrt{2}}$

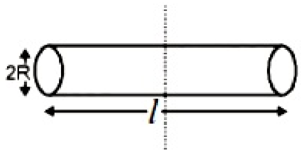
C. $\sqrt{\frac{3}{2}}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Solution:

Solution:
As we know, moment of inertia of a solid cylinder about an axis which is perpendicular bisector



$I = \frac{mR^2}{4} + \frac{ml^2}{12}$

$I = \frac{m}{4}\left[R^2 + \frac{l^2}{3} \right]$

$= \frac{m}{4}\left[\frac{V}{\pi l} + \frac{l^2}{3} \right] \Rightarrow \frac{d l}{d l} = \frac{m}{4}\left[\frac{-V}{\pi l^2} + \frac{2l}{3} \right] = 0$

$\frac{V}{\pi l^2} = \frac{2l}{3} \Rightarrow V = \frac{2\pi l^3}{3}$

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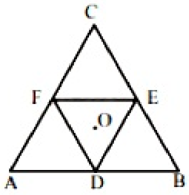
$\pi R^2 l = \frac{2\pi l^3}{3} \Rightarrow \frac{l^2}{R^2} = \frac{3}{2}$ or, $\frac{l}{R} = \sqrt{\frac{3}{2}}$

Question178

Moment of inertia of an equilateral triangular lamina ABC, about the axis passing through its centre O and perpendicular to its plane is I_0 as shown in the figure. A cavity DEF is cut out from the lamina, where D, E, F are the mid points of the sides. Moment of inertia of the remaining part of lamina about the same axis is:



[Online April 8, 2017]

Options:

A. $\frac{7}{8} I_C$

B. $\frac{15}{16} I_o$

C. $\frac{3I_o}{4}$

D. $\frac{31 I_0}{32}$

Answer: B

Solution:

Solution:
According to theorem of perpendicular axes, moment of inertia of triangle (ABC)
$I_0 = kml^2$ ....(i)
$BC = 1$
Moment of inertia of a cavity DEF
$I_{DEF} = K\frac{m}{4}\left(\frac{l}{2}\right)^2$
$= \frac{k}{16} ml^2$
From equation (i),
$I_{DEF} = \frac{I_0}{16}$
Moment of inertia of remaining part
$I_{remain} = I_0 - \frac{I_0}{16} = \frac{15 I_0}{16}$

Question179

A circular hole of radius $\frac{R}{4}$ is made in a thin uniform disc having mass M

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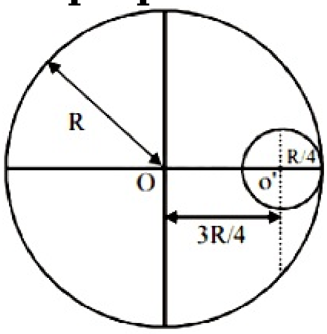
and radius R, as shown in figure. The moment of inertia of the remaining portion of the disc about an axis passing through the point O and perpendicular to the plane of the disc is :



[Online April 9, 2017]

Options:

A. $\frac{219MR^2}{256}$

B. $\frac{237MR^2}{512}$

C. $\frac{19MR^2}{512}$

D. $\frac{197MR^2}{256}$

Answer: B

Solution:

Solution:

Moment of Inertia of complete disc about 'O' point

$I_{total} = \frac{MR^2}{2}$

Radius of removed disc = R / 4

∴ Mass of removed disc = M / 16

[As $M \propto R^2$]

M.I of removed disc about its own axis (O′)

$= \frac{1}{2}\frac{M}{16}\left(\frac{R}{4}\right)^2 = \frac{MR^2}{512}$

M.I of removed disc about O

$I_{removed\ disc} = I_{cm} + mx^2$

$= \frac{MR^2}{512} + \frac{M}{16}\left(\frac{3R}{4}\right)^2 = \frac{19MR^2}{512}$

M.I of remaining disc

$I_{remaining} = \frac{MR^2}{2} - \frac{19}{512}MR^2 = \frac{237}{512}MR^2$

Question180

In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\frac{BC}{AB}$ is close to :

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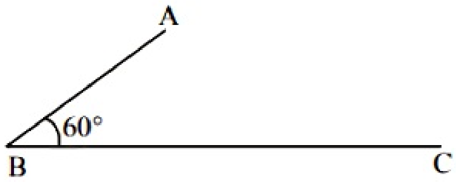


[Online April 10, 2016]

Options:

A. 1.85

B. 1.5

C. 1.37

D. 3

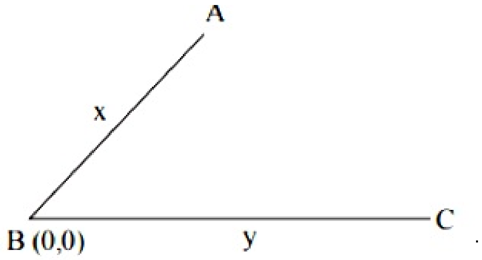
Answer: C

Solution:

Solution:

Centre of mass $x_{cm} = \frac{x}{2} \frac{(\rho x)\left(\frac{x}{2}\right)\frac{1}{2} + \rho y^{y/2}}{\rho(x+y)}$

$\Rightarrow \frac{1}{2} + \frac{y}{x} = \frac{y^2}{x^2}$



$\therefore \frac{Bc}{AB}\frac{y}{x} = \frac{1+\sqrt{3}}{2} = 1.37$

Question181

Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute (rpm) to ensure proper mixing is close to : (Take the radius of the drum to be 1.25m and its axle to be horizontal): [Online April 10, 2016]

Options:

A. 27.0

B. 0.4

C. 1.3

D. 8.0

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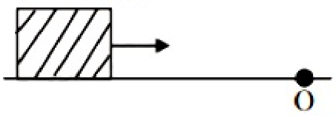
Answer: A

Solution:

Solution:
For just complete rotation
$v = \sqrt{Rg}$ at top point
The rotational speed of the drum
$\Rightarrow \omega = \frac{v}{R} = \sqrt{\frac{g}{R}} = \sqrt{\frac{10}{1.25}}$
The maximum rotational speed of the drum in revolutions per minute
$\omega(\text{rpm}) = \frac{60}{2\pi}\sqrt{\frac{10}{1.25}} = 27$

Question182

A cubical block of side 30cm is moving with velocity $2ms^{-1}$ on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad / s) of the block immediately after it hits the bump, is:



[Online April 9, 2016]

Options:

A. 13.3

B. 5.0

C. 9.4

D. 6.7

Answer: B

Solution:

Solution:
Angular momentum, $mvr = I\,\omega$
Moment of Inertia (I) of cubical block is given by

$$I = m\left(\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right) \therefore \omega = \frac{m \,.\, 2\frac{R}{2}}{m\left[\frac{R^2}{6} + \left(\frac{R}{\sqrt{2}}\right)^2\right]}$$

$$\Rightarrow \omega = 128R = \frac{3}{2 \times 0.3} = \frac{10}{2} = 5\,\text{rad / s}$$

Question183

A particle of mass m is moving along the side of a square of side ′a′,

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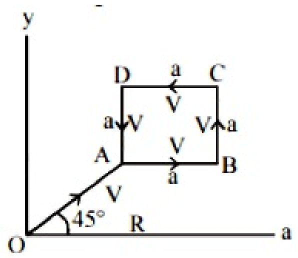
with a uniform speed v in the x − y plane as shown in the figure:



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?
[2016]

Options:

A. $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

B. $L = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A.

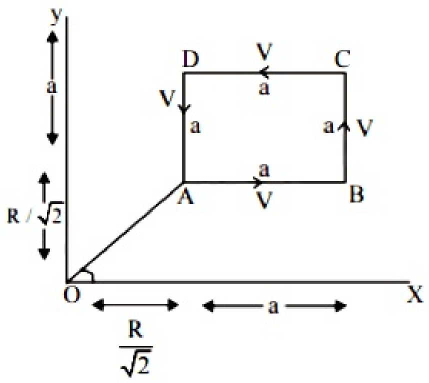
C. $L = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B.

D. $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

Answer: A

Solution:

Solution:
We know that $|L| = mvr_{\perp}$



In none of the cases, the perpendicular distance $r_{\perp}$ is $\left(\frac{R}{\sqrt{2}} + a\right)$

Question184

A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD, which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and Cd (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to:

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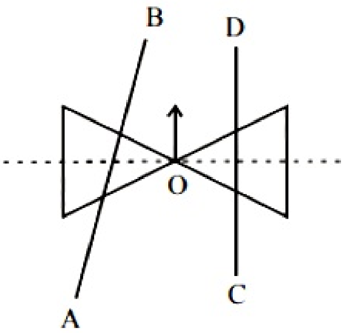


[2016]

Options:

A. go straight.

B. turn left and right alternately.

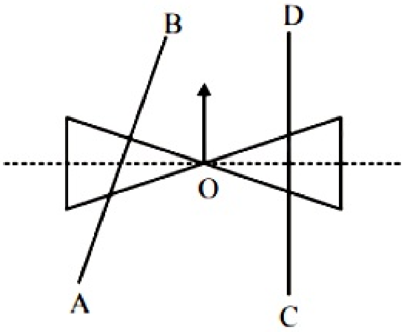
C. turn left.

D. turn right.

Answer: C

Solution:

Solution:
As shown in the diagram, the normal reaction of AB on roller will shift towards O.
This will lead to tending of the system of cones to turn left.



Question185

Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to : [2015]

Options:

A. $\frac{5h}{8}$

B. $\frac{3h^2}{8R}$

C. $\frac{h^2}{4R}$

D. $\frac{3h}{4}$

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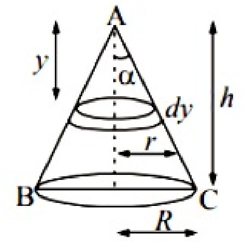
Answer: D

Solution:

Solution:
Let density of cone $= \rho$.

Centre of mass, ycm $= \frac{\int y d m}{\int d m}$

$$= \frac{\int_0^h y \pi r^2 d y \rho}{\frac{1}{3}\pi R^2 h \rho} = \frac{\int_0^h r^2 y d y}{\frac{1}{3} R^2 h} \text{ ....(i)}$$



For a cone we know that

$\frac{r}{R} = \frac{y}{n} \therefore r = \frac{y}{n} R$

$$\text{ycm} = \frac{\int_0^h 3y^3 d y}{h^3} = \frac{3\left[\frac{y^4}{4}\right]_0^h}{h^3} = \frac{3}{4}h$$

Question186

A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)$ L from A, then a and b are related as :
[Online April 11, 2015]

Options:

A. a = 2b

B. 2a = b

C. a = b

D. 3a = 2b

Answer: B

Solution:

Solution:
Centre of mass of the rod is given by:

$$x_{cm} = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right) d x}{\int_0^L \left(a + \frac{bx}{L}\right) d x}$$

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$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$

On solving we get, $b = 2a$

Question187

A particle of mass 2kg is on a smooth horizontal table and moves in a circular path of radius 0.6m. The height of the table from the ground is 0.8m. If the angular speed of the particle is 12rad s^{-1}, the magnitude of its angular momentum about a point on the ground right under the centre of the circle is:
[Online April 11, 2015]

Options:

A. $14.4\text{kgm}^2\text{s}^{-1}$

B. $8.64\text{kgm}^2\text{s}^{-1}$

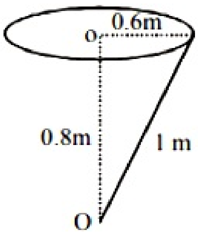
C. $20.16\text{kgm}^2\text{s}^{-1}$

D. $11.52\text{kgm}^2\text{s}^{-1}$

Answer: A

Solution:

Solution:
Angular momentum,
$L_0 = mvr \sin 90°$
$= 2 \times 0.6 \times 12 \times 1 \times 1$
[As $V = r\omega$, $\sin 90° = 1$]
So, $L_0 = 14.4\text{kgm}^2 / \text{s}$



Question188

From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is:
[2015]

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Options:

A. $\frac{4MR^2}{9\sqrt{3}\pi}$

B. $\frac{4MR^2}{3\sqrt{3}\pi}$

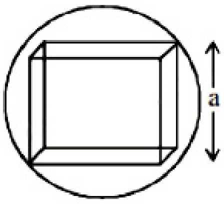
C. $\frac{MR^2}{32\sqrt{2}\pi}$

D. $\frac{MR^2}{16\sqrt{2}\pi}$

Answer: A

Solution:

Solution:



Here $a = \frac{2}{\sqrt{3}}R$

Now, $\frac{M}{M'} = \frac{\frac{4}{3}\pi R^3}{a^3}$

$= \frac{\frac{4}{3}\pi R^3}{\left(\frac{2}{\sqrt{3}}R\right)^3} = \frac{\sqrt{3}}{2}\pi$

$M' = \frac{2M}{\sqrt{3}\pi}$

Moment of inertia of the cube about the given axis,

$I = \frac{M'a^2}{6}$

$= \frac{\frac{2M}{\sqrt{3}\pi} \times \left(\frac{2}{\sqrt{3}}R\right)^2}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$

Question189

Consider a thin uniform square sheet made of a rigid material. If its side is ' a ' mass m and moment of inertia I about one of its diagonals, then :
[Online April 10, 2015]

Options:

A. $I > \frac{ma^2}{12}$

B. $\frac{ma^2}{24} < I < \frac{ma^2}{12}$

C. $I = \frac{ma^2}{24}$

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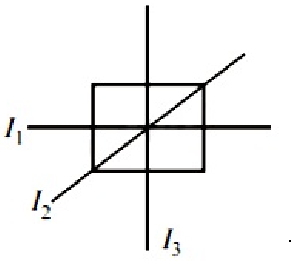
D. $I = \frac{ma^2}{12}$

Answer: D

Solution:

Solution:
For a thin uniform square sheet
$I_1 = I_2 = I_3 = \frac{ma^2}{12}$



Question190

A uniform solid cylindrical roller of mass ' m ' is being pulled on a horizontal surface with force F parallel to the surface and applied at its centre. If the acceleration of the cylinder is 'a' and it is rolling without slipping then the value of 'F' is:
[Online April 10, 2015]

Options:

A. ma

B. $\frac{5}{3}$ma

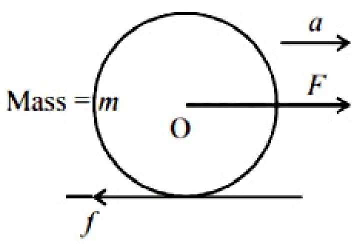
C. $\frac{3}{2}$ma

D. 2ma

Answer: C

Solution:

Solution:
From figure,
$ma = F - f$ ....(i)



And, torque $\tau = I\alpha$
$\frac{mR^2}{2}\alpha = fR$

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$\frac{mR^2}{2}\frac{a}{R} = fR\left[\because \alpha = \frac{a}{R}\right]$

ma2 = f ......(ii)

Put this value in equation (i),

$ma = F - \frac{ma}{2}$ or $F = \frac{3ma}{2}$

Question191

A thin bar of length L has a mass per unit length lambda, that increases linearly with distance from one end. If its total mass is M and its mass per unit length at the lighter end is λ_o, then the distance of the centre of mass from the lighter end is:
[Online April 11, 2014]

Options:

A. $\frac{L}{2} - \frac{\lambda_o L^2}{4M}$

B. $\frac{L}{3} + \frac{\lambda_o L^2}{8M}$

C. $\frac{L}{3} + \frac{\lambda_o L^2}{4M}$

D. $\frac{2L}{3} - \frac{\lambda_o L^2}{6M}$

Answer: C

Solution:

Solution:

Question192

A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed w rad/s about the vertical. About the point of suspension:
[2014]

Options:

A. angular momentum is conserved.

B. angular momentum changes in magnitude but not in direction.

C. angular momentum changes in direction but not in magnitude.

D. angular momentum changes both in direction and magnitude.

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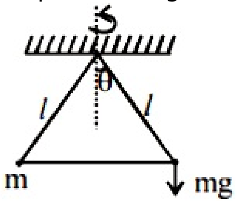
Answer: C

Solution:

Solution:
Torque working on the bob of mass m is, $\tau = mg \times l \sin\theta$. (Direction parallel to plane of rotation of particle)



As τ is perpendicular to $\vec{L}$, direction of L changes but magnitude remains same.

Question193

A ball of mass 160 g is thrown up at an angle of 60° to the horizontal at a speed of 10 ms^{-1}. The angular momentum of the ball at the highest point of the trajectory with respect to the point from which the ball is thrown is nearly ($g = 10\ ms^{-2}$) [Online April 19, 2014]

Options:

A. 1.73 kg m^2/s

B. 3.0 kg m^2/s

C. 3.46 kg m^2/s

D. 6.0 kg m^2/s

Answer: B

Solution:

Solution:
The initial components of velocity are,
$V = 5i + 5 \times 3^{0.5}j$
At the highest point, only the horizontal component will remain.
Hence, $v = 5i$
The vector corresponding to the highest point is given by,
$T = \frac{2u\sin\theta}{g} = 3^{0.5}$
$x = 5 \times \frac{3^{0.5}}{2}$
$y = ut - \frac{1}{2}gt^2$
$y = 7.5 - 3.75 = 3.75$
Hence, the angular momentum is given by the cross product of the radius vector with the velocity vector.
$L = m(\vec{r} \times \vec{v})$
$= 3$

Question194

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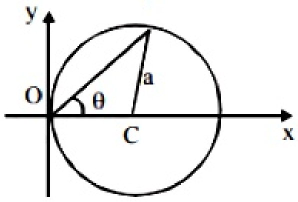
A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The centre of circle is marked by 'C'. The angular momentum from the origin O can be written as:



[Online April 12, 2014]

Options:

A. va (1 + cos 2θ)

B. va (1 + cos θ)

C. va cos 2θ

D. va

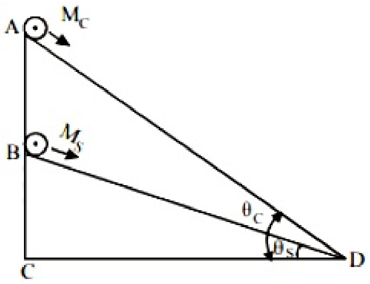
Answer: A

Solution:

Solution:

Question195

A cylinder of mass M_c and sphere of mass M_s are placed at points A and B of two inclines, respectively (See Figure). If they roll on the incline without slipping such that their accelerations are the same, then the ratio $\frac{\sin\theta_c}{\sin\theta_s}$ is:



[Online April 9, 2014]

Options:

A. $\sqrt{\frac{8}{7}}$

B. $\sqrt{\frac{15}{14}}$

C. $\frac{8}{7}$

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D. $\frac{15}{14}$

Answer: D

Solution:

Solution:
As we know,

Acceleration, $a = \dfrac{mg\sin\theta}{m + \dfrac{I}{r^2}}$

For cylinder, $a_c = \dfrac{M_c \cdot g \cdot \sin\theta_c}{M_c + \dfrac{1}{2}\dfrac{M_c R^2}{R^2}} = \dfrac{M_c \cdot g \cdot \sin\theta_c}{M_c + \dfrac{M_c R^2}{2R^2}}$

or, $a_c = \frac{2}{3} g \sin\theta_c$

For sphere,

$a_s = \dfrac{M_s g \sin\theta_s}{M_s + \dfrac{I_s}{r^2}} = \dfrac{M_s g \sin\theta_s}{M_s + \dfrac{2}{5}\dfrac{M R^2}{R^2}}$

or, $a_s = \frac{5}{7} g \sin\theta_s$

given, $a_c = a_s$

i.e., $\frac{2}{3} g \sin\theta_c = \frac{5}{7} g \sin\theta_s$

$\therefore \dfrac{\sin\theta_c}{\sin\theta_s} = \dfrac{\frac{5}{7}g}{\frac{2}{3}g} = \dfrac{15}{14}$

Question196

A boy of mass 20 kg is standing on a 80 kg free to move long cart. There is negligible friction between cart and ground. Initially, the boy is standing 25 m from a wall. If he walks 10 m on the cart towards the wall, then the final distance of the boy from the wall will be
[Online April 23, 2013]

Options:

A. 15 m

B. 12.5 m

C. 15.5 m

D. 17 m

Answer: D

Solution:

Solution:

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Question197

A particle of mass 2kg is moving such that at time t, its position, in meter, is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at t = 2s about the origin in $kgm^{-2}s^{-1}$ is :
[Online April 23, 2013]

Options:

A. $-80\hat{k}$

B. $(10\hat{i} - 16\hat{j})$

C. $-40\hat{k}$

D. $40\hat{k}$

Answer: A

Solution:

Solution:
Angular momentum $L = m(v \times r)$

$= 2kg\left(\frac{dr}{dt} \times r\right) = 2kg\left(4tj \times 5i - 2t^2\hat{j}\right)$

$= 2kg\left(-20t\hat{k}\right) = 2kg \times -20 \times 2m^{-2}s^{-1}\hat{k}$

$= -80\hat{k}$

Question198

A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction.
The angular speed of the door just after the bullet embeds into it will be :
[Online April 9, 2013]

Options:

A. 6.25 rad/sec

B. 0.625 rad/sec

C. 3.35 rad/sec

D. 0.335 rad/sec

Answer: B

Solution:

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Solution:

Question199

A ring of mass M and radius R is rotating about its axis with angular velocity ω. Two identical bodies each of mass m are now gently attached at the two ends of a diameter of the ring. Because of this, the kinetic energy loss will be:
[Online April 25, 2013]

Options:

A. $\frac{m(M+2m)}{M}\omega^2R^2$

B. $\frac{Mm}{(M+m)}\omega^2R^2$

C. $\frac{Mm}{(M+2m)}\omega^2R^2$

D. $\frac{(M+m)M}{(M+2m)}\omega^2R^2$

Answer: C

Solution:

Solution:

Kinetic energy $_{(\text{rotational})} K_R = \frac{1}{2}I\omega^2$

Kinetic energy $_{(\text{translational})} K_T = \frac{1}{2}Mv^2 \quad (v = R\omega)$

M.I. $_{(\text{initial})} I_{\text{ring}} = MR^2; \omega_{\text{initial}} = \omega$

M.I. $_{(\text{new})} I'_{(\text{system})} = MR^2 + 2mR^2$

$\omega'_{(\text{system})} = \frac{M\omega}{M+2m}$

Solving we get loss in $K.E.$

$= \frac{Mm}{(M+2m)}\omega^2R^2$

Question200

A loop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?
[2013]

Options:

A. $\frac{r\omega_0}{4}$

B. $\frac{r\omega_0}{3}$

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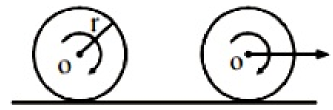
C. $\frac{r\omega_0}{2}$

D. $r\omega_0$

Answer: C

Solution:

Solution:

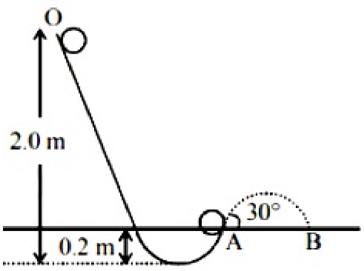


From conservation of angular momentum about any fix point on the surface,
$mr^2\omega_0 = 2mr^2\omega$

$\Rightarrow \omega = \omega_0 / 2 \Rightarrow v = \frac{\omega_0 r}{2} [\because v = r\omega]$

Question201

A tennis ball (treated as hollow spherical shell) starting from O rolls down a hill. At point A the ball becomes air borne leaving at an angle of 30° with the horizontal. The ball strikes the ground at B. What is the value of the distance AB?
(Moment of inertia of a spherical shell of mass m and radius R about its diameter $= \frac{2}{3}mR^2$)



[Online April 22, 2013]

Options:

A. 1.87m

B. 2.08m

C. 1.57m

D. 1.77m

Answer: B

Solution:

Solution:
Velocity of the tennis ball on the surface of the earth or ground

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$v = \sqrt{\dfrac{2gh}{1 + \dfrac{k^2}{R^2}}}$ (where $k =$ radius of gyration of spherical shell $= \sqrt{\frac{2}{3}}R$)

Horizontal range $AB = \dfrac{v^2 \sin 2\theta}{g}$

$= \dfrac{\left(\sqrt{\dfrac{2gh}{1 + k^2/R^2}}\right)^2 \sin(2 \times 30°)}{g} = 2.08m$

Question202

Two point masses of mass $m_1 = fM$ and $m_2 = (1 - f)M$ ($f < 1$) are in outer space (far from gravitational influence of other objects) at a distance R from each other. They move in circular orbits about their centre of mass with angular velocities ω_1 for m_1 and ω_2 for m_2. In that case
[Online May 19, 2012]

Options:

A. $(1 - f)\omega_1 = f\omega$

B. $\omega_1 = \omega_2$ and independent of f

C. $f\omega_1 = (1 - f)\omega_2$

D. $\omega_1 = \omega_2$ and depend on f

Answer: B

Solution:

Solution:
Angular velocity is the angular displacement per unit time i.e., $\omega = \dfrac{\Delta\theta}{\Delta t}$
Here $\omega_1 = \omega_2$ and independent of f.

Question203

A stone of mass m, tied to the end of a string, is whirled around in a circle on a horizontal frictionless table. The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle. The value of n is equal to
[Online May 26, 2012]

Options:

A. – 1

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B. – 2

C. – 4

D. – 3

Answer: D

Solution:

Solution:

Question204

This question has Statement land Statement 2 . Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement 1: When moment of inertia I of a body rotating about an axis with angular speed ω increases, its angular momentum L is unchanged but the kinetic energy K increases if there is no torque applied on it.
Statement 2: $L = I\omega$, kinetic energy of rotation $= \frac{1}{2} I \omega^2$
[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

B. Statement 1 is false, Statement 2 is true.

C. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation of the Statement 1

D. Statement 1 is true, Statement 2 is false.

Answer: B

Solution:

Solution:
As $L = I\omega$ so L increases with increase in omega. Kinetic energy $_{(rotational)}$ depends on an angular velocity ' ω ' and moment of inertia of the body I .
i.e., $K.E._{(rotational)} = \frac{1}{2} I \omega^2$

Question205

A solid sphere having mass m and radius r rolls down an inclined plane. Then its kinetic energy is
[Online May 7, 2012]

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Options:

A. $\frac{5}{7}$ rotational and $\frac{2}{7}$ translational

B. $\frac{2}{7}$ rotational and $\frac{5}{7}$ translational

C. $\frac{2}{5}$ rotational and $\frac{3}{5}$ translational

D. $\frac{1}{2}$ rotational and $\frac{1}{2}$ translational

Answer: B

Solution:

Solution:

$K.E_{\text{rotational}} = \frac{1}{2} I \omega^2$

$= \frac{1}{2}\frac{2}{5}\omega r^2 d^2 \left(\because I_{\text{Solid sphere}} = \frac{2}{5} m r^2 \right)$

$K.E_{\text{translational}} = \frac{1}{2} m v^2$

$\therefore \frac{K.E_{\text{rotational}}}{K.E_{\text{translational}}} = \frac{2}{5}$

Hence option (b) is correct

Question206

A circular hole of diameter R is cut from a disc of mass M and radius R; the circumference of the cut passes through the centre of the disc. The moment of inertia of the remaining portion of the disc about an axis perpendicular to the disc and passing through its centre is [Online May 7, 2012]

Options:

A. $\left(\frac{15}{32}\right) M R^2$

B. $\left(\frac{1}{8}\right) M R^2$

C. $\left(\frac{3}{8}\right) M R^2$

D. $\left(\frac{13}{32}\right) M R^2$

Answer: D

Solution:

Solution:

M.I. of complete disc about its centre O.

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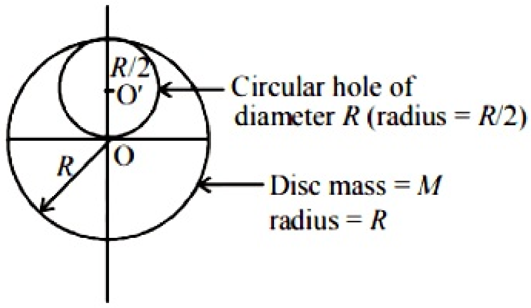
$I_{Total} = \frac{1}{2}MR^2$ ....(i)



Mass of circular hole (removed)

$= \frac{M}{4}\left(\text{As } M = \pi R^2 t \therefore M \propto R^2\right)$

M.I. of removed hole about its own axis

$= \frac{1}{2}\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{1}{32}MR^2$

M.I. of removed hole about O′

$I_{removed\ hole} = I_{cm} + mx^2$

$= \frac{MR^2}{32} + \frac{M}{4}\left(\frac{R}{2}\right)^2$

$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$

M.I. of complete disc can also be written as

$I_{Total} = I_{removed\ hole} + I_{remaining\ disc}$

$I_{Total} = \frac{3MR^2}{32} + I_{remaining\ disc}$ ....(ii)

From eq. (i) and (ii),

$\frac{1}{2}MR^2 = \frac{3MR^2}{32} + I_{remaining\ disc}$

$\Rightarrow I_{remaining\ disc} = \frac{MR^2}{2} - \frac{3MR^2}{32} = \left(\frac{13}{32}\right)MR^2$

Question207

A thick-walled hollow sphere has outside radius R_0. It rolls down an incline without slipping and its speed at the bottom is v_0. Now the incline is waxed, so that it is practically frictionless and the sphere is observed to slide down (without any rolling). Its speed at the bottom is observed to be $5v_0 / 4$. The radius of gyration of the hollow sphere about an axis through its centre is
[Online May 26, 2012]

Options:

A. $3R_0 / 2$

B. $3R_0 / 4$

C. $9R_0 / 16$

D. $3R_0$

Answer: B

Solution:

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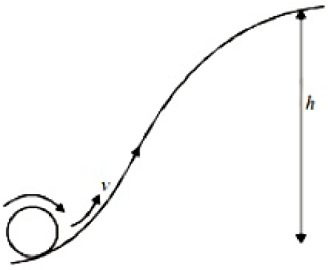
Solution:
When body rolls dawn on inclined plane with velocity V_0 at bottom then body has both rotational and translational kinetic energy.
Therefore, by law of conservation of energy,
$P.E. = K.E_{trans} + K.E_{rotational}$
$= \frac{1}{2}mV_0^2 + \frac{1}{2}I\omega^2$
$= \frac{1}{2}mV_0^2 + \frac{1}{2}mk^2\frac{V_0^2}{R_0^2}$ ......(i) $\left[\because I = mk^2, \omega = \frac{V}{R_0}\right]$
When body is sliding down then body has only translatory motion.
$\therefore P.E. = K.E_{trans}$
$= \frac{1}{2}m\left(\frac{5}{4}v_0\right)^2$ ......(ii)
Dividing (i) by (ii) we get
$\frac{P.E}{P.E} = \frac{\frac{1}{2}mv_0^2\left[1 + \frac{K^2}{R_0^2}\right]}{\frac{1}{2} \times \frac{25}{16} \times mV_0^2} = \frac{25}{16} = 1 + \frac{K^2}{R_0^2} \Rightarrow \frac{K^2}{R_0^2} = \frac{9}{16}$
or, $K = \frac{3}{4}R_0$

Question208

A solid sphere is rolling on a surface as shown in figure, with a translational velocity vms^{-1}. If it is to climb the inclined surface continuing to roll without slipping, then minimum velocity for this to happen is



[Online May 12, 2012]

Options:

A. $\sqrt{2gh}$

B. $\sqrt{\frac{7}{5}gh}$

C. $\sqrt{\frac{7}{2}gh}$

D. $\sqrt{\frac{10}{7}gh}$

Answer: D

Solution:

Solution:
Minimum velocity for a bodyrolling without slipping

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$$v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}}$$

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

$$\therefore v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}} = \sqrt{\frac{10}{7}gh}$$

Question209

A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc.
[2011]

Options:

A. continuously decreases

B. continuously increases

C. first increases and then decreases

D. remains unchanged

Answer: C

Solution:

Solution:
Angular momentum, $L = I\omega \Rightarrow L = mr^2\omega$
As insect moves along a diameter, the effective mass and hence moment of inertia (I) first decreases then increases so from principle of conservation of angular momentum, angular speed omega first increases then decreases.

Question210

A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is:
[2011]

Options:

A. g

B. $\frac{2}{3}g$

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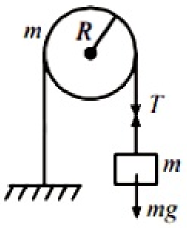
C. $\frac{g}{3}$

D. $\frac{3}{2}g$

Answer: B

Solution:

Solution:
For translational motion,
$mg - T = ma$ ......(i)
For rotational motion, $T \cdot R = I\alpha$



$\Rightarrow T \cdot R = \frac{1}{2}mR^2\alpha$
Also, acceleration, $a = R\alpha$
$\therefore T = \frac{1}{2}mR\alpha = \frac{1}{2}ma$
Substituting the value of T is equation (1) we get $mg - \frac{1}{2}ma = ma \Rightarrow a = \frac{2}{3}g$

Question211

A pulley of radius 2m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is $10 kg - m^2$ the number of rotations made by the pulley before its direction of motion is reversed, is:
[2011]

Options:

A. more than 3 but less than 6

B. more than 6 but less than 9

C. more than 9

D. less than 3

Answer: A

Solution:

Solution:
Given,
Force, $F = (20t - 5t^2)$
Radius, $r = 2m$
Torque, $T = rf = I\alpha$
$\Rightarrow 2(20t - 5t^2) = 10\alpha$

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$\therefore \alpha = 4t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^{t} (4t - t^2) dt$

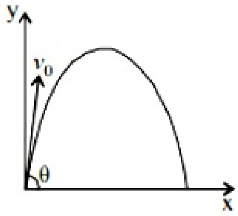
$\Rightarrow \omega = 2t^2 - \frac{t^3}{3} \left(\text{as } \omega = 0 \text{ at } t = 0, 6s \right)$

$\int_0^{\theta} d\theta = \int_0^{6} \left(2t^2 - \frac{t^3}{3} \right) dt$

$\Rightarrow \theta = 36 rad \Rightarrow 2\pi n = 36 \Rightarrow n = \frac{36}{2\pi} < 6$

Question212

A small particle of mass m is projected at an angle θ with the x -axis with an initial velocity v_0 in the x -y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



[2010]

Options:

A. $-mgv_0 t^2 \cos\theta \, \hat{j}$

B. $mgv_0 t \cos\theta \, \hat{k}$

C. $-\frac{1}{2} mgv_0 t^2 \cos\theta \, \hat{k}$

D. $\frac{1}{2} mgv_0 t^2 \cos\theta \, \hat{i}$

Answer: C

Solution:

Solution:

$\vec{L} = m(\vec{r} \times \vec{v})$

$\vec{L} = m\left[v_0 \cos\theta \, t \, \hat{i} + \left(v_0 \sin\theta \, t - \frac{1}{2} g t^2 \right) \hat{j} \right] \times \left[v_0 \cos\theta \, \hat{i} + (v_0 \sin\theta - gt) \hat{j} \right]$

$= mv_0 \cos\theta \, t \left[-\frac{1}{2} gt \right] \hat{k}$

$= -\frac{1}{2} mgv_0 t^2 \cos\theta \, \hat{k}$

Question213

A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is

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ω. Its centre of mass rises to a maximum height of
[2009]

Options:

A. $\frac{1}{6}\frac{l\omega}{g}$

B. $\frac{1}{2}\frac{l^2\omega^2}{g}$

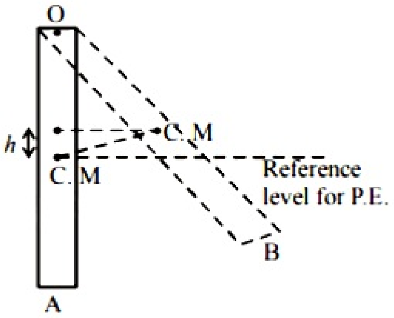
C. $\frac{1}{6}\frac{l^2\omega^2}{g}$

D. $\frac{1}{3}\frac{l^2\omega^2}{g}$

Answer: C

Solution:

Solution:



The moment of inertia of the rod about O is $\frac{1}{3}ml^2$. The kinetic energy of the rod at position A = $\frac{1}{2}I\omega^2$ where I is the moment of inertia of the rod about O. When the rod is in position B, its angular velocity is zero. In this case, the energy of the rod is mgh where h is the maximum height to which the centre of mass (C.M) rises

Gain in potential energy = Loss in kinetic energy

$\therefore mgh = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$

$\Rightarrow h = \frac{l^2\omega^2}{6g}$

Question214

A thin rod of length ' L ' is lying along the x -axis with its ends at x = 0 and x = L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against ' n, which of the following graphs best approximates the dependence of x_{CM} on n ?
[2008]

Options:

A.

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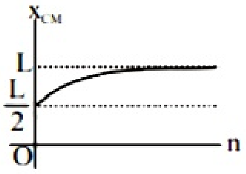
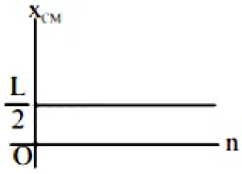
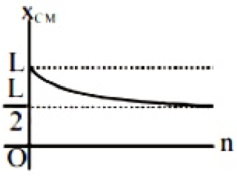


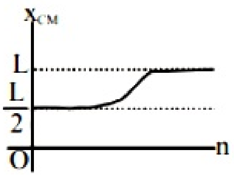
B.



C.



D.



Answer: A

Solution:

Solution:

The linear mass density $\lambda = k\left(\frac{x}{L}\right)^n$ Here $\frac{x}{L} \leq 1$

With increase in the value of n, the centre of mass shift towards the end $x = L$ This is satisfied by only option (a).

$$x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n x\,dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx}$$

$$= \frac{k\left[\frac{x^{n+2}}{(n+2)L^n}\right]_0^L}{\left[\frac{kx^{n+1}}{(n+1)L^n}\right]_0^L} = \frac{L(n+1)}{n+2}$$

For $n = 0$, $x_{CM} = \frac{L}{2}$; $n = 1$,

$x_{CM} = \frac{2L}{3}$; $n = 2$, $x_{CM} = \frac{3L}{4}$; ....

For $n \to \infty x_{cm} = L$

Moment of inertia of a square plate about an axis through its centre and perpendicular to its plane is.

Question215

Consider a uniform square plate of side ' a ' and mass ' M ' The moment of inertia of this plate about an axis perpendicular to its plane and

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passing through one of its corners is
[2008]

Options:

A. $\frac{5}{6}M a^2$

B. $\frac{1}{12}M a^2$

C. $\frac{7}{12}M a^2$

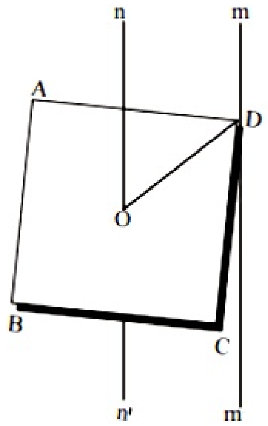
D. $\frac{2}{3}M a^2$

Answer: D

Solution:

Solution:

$I_{nn'} = \frac{1}{12}M(a^2 + a^2) = \frac{M a^2}{6}$



Also, $DO = \frac{DB}{2} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

By parallel axes the orem, moment of inertia of plate about an axis through one of its corners.

$$I_{mm'} = I_{nn'} + M\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{M a^2}{6} + \frac{M a^2}{2}$$

$$= \frac{M a^2 + 3M a^2}{6} = \frac{2}{3}M a^2$$

Question216

A circular disc of radius R is removed from a bigger circular disc of radius 2R such that the circumferences of the discs coincide. The centre of mass of the new disc is α/R form the centre of the bigger disc. The value of α is
[2007]

Options:

A. 1/4

B. 1/3

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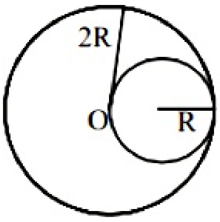
C. 1/2

D. 1/6

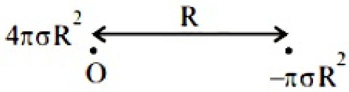
Answer: B

Solution:

Solution:
Let σ be the mass per unit area of the disc.
Then the mass of the complete disc $= \sigma(\pi(2R)^2)$



The mass of the removed disc $= \sigma(\pi R^2) = \pi\sigma R^2$
Let us consider the above situation to be a complete disc of radius $2R$ on which a disc of radius R of negative mass is superimposed. Let O be the origin. Then the above figure can be redrawn keeping in mind the concept of centre of mass as :



$$x_{c.m} = \frac{(6\pi(2R)^2) \times 0 + (-6(\pi R^2))R}{4\pi\sigma R^2 - \pi\sigma R^2}$$

$$\therefore x_{c.m} = \frac{-\pi\sigma R^2 \times R}{3\pi\sigma R^2}$$

$$\therefore x_{c.m} = -\frac{R}{3} = \alpha R \Rightarrow \alpha = \frac{1}{3}$$

Question217

Angular momentum of the particle rotating with a central force is constant due to [2007]

Options:

A. constant torque

B. constant force

C. constant linear momentum

D. zero torque

Answer: D

Solution:

Solution:

We know Torque $\vec{\tau}_c = \frac{d\vec{L}_c}{dt}$ where

$\vec{L}_c =$ Angular momentum about the center of mass of the body. Central forces act along the center of mass.
Therefore torque about center of mass is zero.

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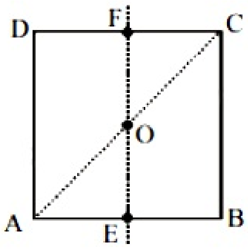
$\therefore \tau = \frac{dL}{dt} = 0 \Rightarrow \vec{L}_c = \text{constt.}$

Question218

For the given uniform square lamina ABCD, whose centre is O



[2007]

Options:

A. $I_{AC} = \sqrt{2} I_{EF}$

B. $\sqrt{2} I_{AC} = I_{EF}$

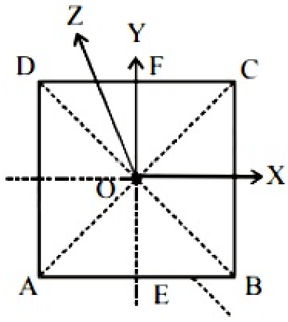
C. $I_{AD} = 3 I_{EF}$

D. $I_{AC} = I_{EF}$

Answer: D

Solution:

Solution:
By the theorem of perpendicular axes,
$I = I_{EF} + I_{GH}$
Here, I is the moment of inertia of square lamina about an axis through O and perpendicular to its plane.
$\therefore I_{EF} = I_{GH}$ (By Symmetry of Figure)



$\therefore I_{EF} = \frac{I}{2}$ ......(i)
Again, by the same theorem $I = I_{AC} + I_{BD} = 2I_{AC}$
($\therefore I_{AC} = I_{BD}$ by symmetry of the figure)
$\therefore I_{AC} = \frac{I}{2}$ ....(ii)
From (i) and (ii), we get, $I_{EF} = I_{AC}$

Question219

A round uniform body of radius R, mass M and moment of inertia I

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rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is [2007]

Options:

A. $\frac{g \sin \theta}{1 - M R^2 / I}$

B. $\frac{g \sin \theta}{1 + I / M R^2}$

C. $\frac{g \sin \theta}{1 + M R^2 / I}$

D. $\frac{g \sin \theta}{1 - I / M R^2}$

Answer: B

Solution:

Solution:
Acceleration of the body rolling down an inclined plane is given by.

$a = \frac{g \sin \theta}{1 + \frac{I}{M R^2}}$

Question220

Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d , by what distance should the second particle is moved, so as to keep the centre of mass at the same position? [2006]

Options:

A. $\frac{m_2}{m_1} d$

B. $\frac{m_1}{m_1 + m_2} d$

C. $\frac{m_1}{m_2} d$

D. d

Answer: C

Solution:

Solution:

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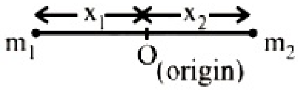
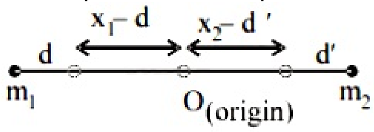
Initially, 

$0 = \frac{m_1(-x_1) + m_2x_2}{m_1 + m_2} \Rightarrow m_1x_1 = m_2x_2$ .... (1)

Let the particles is displaced through distanced away from centre of mass



$\therefore 0 = \frac{m_1(d - x_1) + m_2(x_2 - d')}{m_1 + m_2}$

$\Rightarrow 0 = m_1d - m_1x_1 + m_2x_2 - m_2d'$

$\Rightarrow d' = \frac{m_1}{m_2}d$ [From(1).]

Question221

A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' =
[2006]

Options:

A. $\frac{\omega(m + 2M)}{m}$

B. $\frac{\omega(m - 2M)}{(m + 2M)}$

C. $\frac{\omega m}{(m + M)}$

D. $\frac{\omega m}{(m + 2M)}$

Answer: D

Solution:

Solution:
Applying conservation of angular momentum $I'\omega' = I\omega$
$(mR^2 + 2MR^2)\omega' = mR^2\omega$
$\Rightarrow (m + 2m)R^2\omega' = mR^2\omega$
$\Rightarrow \omega' = \omega\left[\frac{m}{m + 2M}\right]$

Question222

A force of $-F\hat{k}$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is

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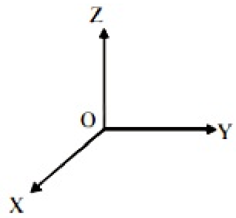

[2006]

Options:

A. $F(\hat{i} - \hat{j})$

B. $-F(\hat{i} + \hat{j})$

C. $F(\hat{i} + \hat{j})$

D. $-F(\hat{i} - \hat{j})$

Answer: C

Solution:

Solution:

Torque $\vec{\tau} = \vec{r} \times \vec{F} = (\hat{i} - \hat{j}) \times (-F\hat{k})$

$= F[-\hat{i} \times \hat{k} + \hat{j} \times \hat{k}] = F(\hat{j} + \hat{i}) = F(\hat{i} + \hat{j})$

[Since $\hat{k} \times \hat{i} = \hat{j}$ and $\hat{j} \times \hat{k} = \hat{i}$]

Question223

Four point masses, each of value m, are placed at the corners of a square ABCD of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is
[2006]

Options:

A. $2ml^2$

B. $\sqrt{3}ml^2$

C. $3ml^2$

D. ml^2

Answer: C

Solution:

Solution:

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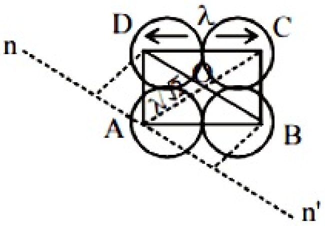

$I_{nn'}$ = M . I due to the point mass at B+
M . I due to the point mass at D+
M . I due to the point mass at C.

$I_{nn'} = m\left(\frac{l}{\sqrt{2}}\right)^2$
$+m\left(\frac{l}{\sqrt{2}}\right)^2$
$+m(\sqrt{2}l)^2$

$\Rightarrow I_{nn'} = 2 \times m\left(\frac{l}{\sqrt{2}}\right)^2 + m(\sqrt{2}l)^2$

$= ml^2 + 2ml^2 = 3ml^2$

Question224

A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}$ M and a body C of mass $\frac{2}{3}$M . The centre of mass of bodies B and C taken together shifts compared to that of body A towards
[2005]

Options:

A. does not shift

B. depends on height of breaking

C. body B

D. body C

Answer: A

Solution:

Solution:
The centre of mass of bodies B and C taken together does not shift as no external force acts. The centre of mass of the system continues its original path. It is only the internal forces which comes into play while breaking.

Question225

A ' T shaped object with dimensions shown in the figure is lying on a smooth floor. A force ' $\vec{F}$ ' is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.

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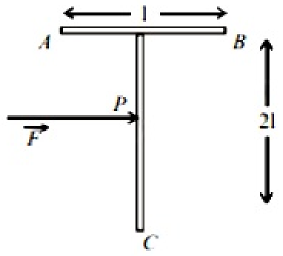


[2005]

Options:

A. $\frac{3}{2}l$

B. $\frac{2}{3}l$

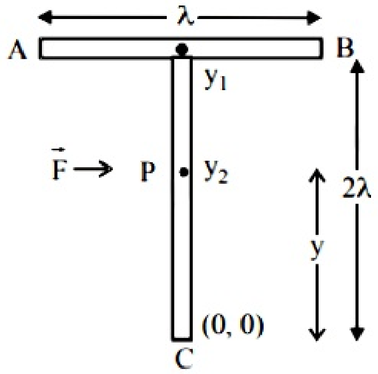
C. l

D. $\frac{4}{3}l$

Answer: D

Solution:

Solution:



To have translational motion without rotation, the force $\vec{F}$ has to be applied at centre of mass. i.e. the point ' P 'has to be at the centre of mass

Taking point C at the origin position, positions of y, and y_2 are $r_1 = 2l$, $r_2 = l$ and $ml = m$ and $m_2 = 2m$

$$y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m \times 2l + 2m \times l}{3m} = \frac{4l}{3}$$

Question226

The moment of inertia of a uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is
[2005]

Options:

A. $\frac{2}{5}M r^2$.

B. $\frac{1}{4}M r$

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C. $\frac{1}{2}M r^2$

D. $M r^2$

Answer: C

Solution:

Solution:
The disc may be assumed as combination of two semi circular parts. Therefore, circular disc will have twice the mass of semicircular disc.

Moment of inertia of disc = $\frac{1}{2}(2m)r^2 = M r^2$

Let I be the moment of inertia of the uniform semicircular disc

$\Rightarrow 2I = 2M r^2 \Rightarrow I = \frac{M r^2}{2}$

Question227

A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected ?
[2004]

Options:

A. Angular velocity

B. Angular momentum

C. Moment of inertia

D. Rotational kinetic energy

Answer: B

Solution:

Solution:
Angular momentum will remain the same since no external torque act in free space.

Question228

One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B Such that
[2004]

Options:

A. $I_A < I_B$

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B. $I_A > I_B$

C. $I_A = I_B$

D. $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

Answer: A

Solution:

Solution:

The moment of inertia of solid sphere A about its diameter $I_A = \frac{2}{5} M R^2$.

The moment of inertia of a hollow sphere B about its diameter $I_B = \frac{2}{3} M R^2$.

$\therefore I_A < I_B$

Question229

Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$, and $\vec{T}$ be the torque of this force about the origin. Then
[2003]

Options:

A. $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} \neq 0$

B. $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} = 0$

C. $\vec{r} \cdot \vec{T} \neq 0$ and $\vec{F} \cdot \vec{T} \neq 0$

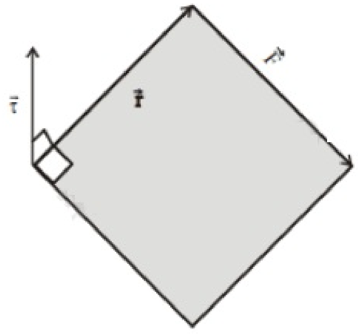
D. $\vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$

Answer: D

Solution:

Solution:

We know that $\vec{\tau} = \vec{r} \times \vec{F}$



Vector $\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$. We also know that the dot product of two vectors which have an angle of 90° between them is zero.

$\therefore \vec{r} \cdot \vec{T} = 0$ and $\vec{F} \cdot \vec{T} = 0$

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Question230

A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius 4R is made from an iron plate of thickness $\frac{t}{4}$. Then the relation between the moment of inertia I_X and I_I is [2003]

Options:

A. $I_Y = 32I_X$

B. $I_Y = 16I_X$

C. $I_Y = I_X$

D. $I_Y = 64I_X$

Answer: D

Solution:

Solution:

We know that density $(d) = \frac{mass(M)}{volume(V)}$

$\therefore M = d \times V = d \times (\pi R^2 \times t)$

The moment of inertia of a disc is given by $I = \frac{1}{2}MR^2$

$\therefore I_x = \frac{1}{2}M_x R_x^{\ 2} = \frac{1}{2}(d \times \pi R^2 \times t)R^2$

$= \frac{\pi d}{2} t \times R^4$

$I_y = \frac{1}{2}M_y R_y^{\ 2} = \frac{1}{2}\left[\pi(4R^2)\left(\frac{1}{4}\right)d\right] \times (4R)^2$

$\therefore \frac{I_X}{I_Y} = \frac{t_X R_X^{\ 4}}{t_Y R_Y^{\ 4}} = \frac{t \times R^4}{\frac{t}{4} \times (4R)^4} = \frac{1}{64}$

Question231

A particle performing uniform circular motion has angular frequency is doubled & its kinetic energy halved, then the new angular momentum is [2003]

Options:

A. $\frac{L}{4}$

B. 2L

C. 4L

D. $\frac{L}{2}$

Answer: A

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Solution:

Solution:

Rotational kinetic energy $= \frac{1}{2} I \omega^2$,

Angular momentum, $L = I\omega \Rightarrow I = \frac{L}{\omega}$

$\therefore K.E. = \frac{1}{2}\frac{L}{\omega} \times \omega^2 = \frac{1}{2} L\omega$

$L' = \frac{2K.E}{\omega}$

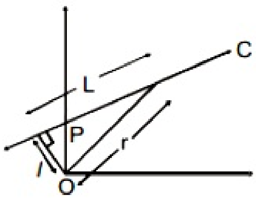
When ω is doubled and $K.E$ is haled New angular momentum,

$L' = \frac{\frac{2K.E}{2}}{2\omega}$

$\Rightarrow \therefore L' = \frac{L}{4}$

Question232

A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about P?



[2002]

Options:

A. mvL

B. mvl

C. mvr

D. zero.

Answer: D

Solution:

Solution:
Angular momentum (L)
= (linear momentum) × (perpendicular distance of the line of action of momentum from the axis of rotation)
As the particle moves with velocity V along line PC, the line of motion passes through P.
$\therefore L = mv \times r$
$= mv \times 0$
$= 0$

Question233

Moment of inertia of a circular wire of mass M and radius R about its diameter is

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[2002]

Options:

A. $M R^2 / 2$

B. $M R^2$

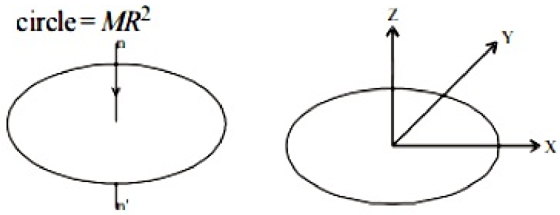
C. $2M R^2$

D. $M R^2 / 4$

Answer: A

Solution:

Solution:
M. I of a circular wire about an axis nn' passing through the centre of the circle and perpendicular to the plane of the circle $= M R^2$



As shown in the figure, X -axis and Y -axis lie along diameter of the ring. Using perpendicular axis theorem
$I_X + I_Y = I_Z$
Here, I_X and I_Y are the moment of inertia about the diameter.
$\Rightarrow 2I_X{}^A = M R^2 [\because I_X = I_Y$ (by symmetry) and $I_Z = M R^2]$
$\therefore I_X = \frac{1}{2} M R^2$

Question234

Initial angular velocity of a circular disc of mass M is ω_1. Then two small spheres of mass m are attached gently to diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc?
[2002]

Options:

A. $\left(\frac{M + m}{M}\right) \omega_1$

B. $\left(\frac{M + m}{m}\right) \omega_1$

C. $\left(\frac{M}{M + 4m}\right) \omega_1$

D. $\left(\frac{M}{M + 2m}\right) \omega_1$

Answer: C

Solution:

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Moment of inertia of circular disc $I_1 = \frac{1}{2} M R^2$

When two small sphere are attached on the edge of the disc, the moment of inertia becomes

$I_2 = \frac{1}{2} M R^2 + 2mR^2$

When two small spheres of mass m are attached gently, the external torque, about the axis of rotation, is zero and therefore the angular momentum about the axis of rotation is constant.

$\therefore I_1 \omega_1 = I_2 \omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2} \omega_1$

$$\therefore \omega_2 = \frac{\frac{1}{2} M R^2}{\frac{1}{2} M R^2 + 2mR^2} \times \omega_1 = \frac{M}{M + 4m} \omega_1$$

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